

Low-energy radiative-capture reactions within two-cluster coupled-channel description

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Abstract

The formalism that describes radiative-capture reactions at low energies within an extended two-cluster potential model is presented. Construction of the operator of single-photon emission is based on a generalisation of the Siegert theorem with which the amplitude of the electromagnetic process is constructed in an explicitly gauge-independent way. While the starting point for this construction is a microscopic (single-nucleon) current model, the resulting operator of low-energy photon emission by a two-cluster system is expressed in terms of macroscopic quantities for the clusters and does not depend directly on their intrinsic coordinates and momenta. The multichannel algebraic scattering (MCAS) approach has been used to construct the initial- and final-state wave functions. We present a general expression for the scattering wave function obtained from the MCAS T matrix taking into account inelastic channels and Coulomb distortion. The developed formalism has been tested on the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction cross section at astrophysical energies. The energy dependence of the evaluated cross section and S factor agrees well with that extracted from measurement though the calculated quantities slightly overestimate data.

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1 Introduction

In spite of several decades of intensive experimental and theoretical studies of the radiative capture of nuclei at low-energies, reactions of this type remain a focus of research activities. One of the main reasons for this continued and growing interest in this kind of nuclear

processes is their importance in understanding the Big Bang nucleo-synthesis and stellar evolution (particularly processes that occur in the interior of the Sun). Another aspect of astrophysical relevance that requires improved knowledge of the stellar fusion reactions, is the solar neutrino flux problem. In particular, a recent assessment [1] of the current understanding of solar neutrino fluxes called for new measurements of the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction cross section at very low (solar) energies with an accuracy of better than $\pm 5\%$. Of course, it is a serious challenge for an experiment measuring cross sections at low energies to reach such a precision since the reaction yield rate is extremely small due to the strong Coulomb repulsion.

Some current challenges for nuclear astrophysics can be met with new experimental facilities such as the ones at the Laboratory for Underground Nuclear Astrophysics (LUNA) [2] that is part of Italy's Gran Sasso National Laboratory (LNGS). The LUNA collaboration has reported on their first measurements [3] of the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ total cross section at very low (down to 127 keV) center-of-mass (c.m.) energies with a total uncertainty of 4%. High-precision measurements of cross sections for other radiative-capture reactions of astrophysical relevance are under way [4]. However, some processes are, and will remain the province of purely theoretical studies. Nonetheless, the impressive progress achieved in recent times in the experimental studies of low-energy radiative-capture reactions is an additional argument for boosting research activities on the theoretical side as well.

The photon-emission process relevant to stellar fusion reactions is known [5] to occur dominantly at a large (extra-nuclear) distance. In general, then, one may assume that no intermediate “compound” nuclear system is formed. If that is so, the process is defined as a direct-capture reaction. The ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction, which provides termination of the proton-proton stellar chain through ${}^3\text{He}$ burning, is a one case where successful application of the direct-capture model as a description of the photon emission process has been made. One of the earliest detailed analysis of this reaction, and of the mirror process ${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$, assuming a two-body direct-capture picture was made in Ref. [6]. In the calculations of the total cross section and of the astrophysical S factor, the authors of that paper used a bound-state radial wave function taken in the asymptotic (Whittaker) form while a repulsive nuclear hard-core model that took into account Coulomb distortion was used to describe the initial (scattering) state. Such a choice for the initial-state model was favoured by the analysis of the then existing experimental data for elastic α scattering from the $A = 3$ nucleus and the phase shifts deduced therefrom. It was shown many years later [7] that this repulsive effect can also result from inner oscillation in the overlap with the relative wave function of the two nuclei. Those later calculations were made using both a two-body potential model and the orthogonality condition model (OCM) [8] to construct the wave function.

The direct nature of many of stellar reactions suggests that using a cluster model may be an appropriate approach for their description. In general, a great amount of experimental information on these reactions has been accumulated and extensive cluster-model studies of the stellar fusion processes have been made [9]. Methodologically, studies have ranged from the potential cluster models [10, 11] which consider the two clusters as point-like objects in a two-body potential, to microscopic approaches which include the resonating-group method (RGM) [12] as well as the generator coordinate method [13] and the OCM [8], both of which are closely related to the RGM.

In addition to the cluster models, the variational Monte Carlo technique has been

elaborated [14] and applied in studies of few-nucleon systems. The dynamical input in this approach is given by microscopic (nucleon-nucleon) interactions. Doing so provides a universality of description and reduces the number of model parameters. In an application of this approach [15], total cross sections for the reactions ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ and ${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$ from calculations were found. But this technique could not reproduce adequately nuclear binding energies, and the states of ${}^7\text{Be}$ were so under-bound as to be nonphysical. On the other hand, microscopic grounds to justify use of potential cluster models have been established within the so-called microscopic potential model [9]. Possible extensions of the two-cluster model space were also studied within that approach (see Ref. [16]).

Nevertheless, many of essential dynamical features of low-energy radiative capture processes with light nuclei are still missing in the existing theoretical models. One of the important issues that has not been properly addressed so far is the role in the EM process of collective (rotational and vibrational) degrees of freedom of the colliding nuclei. This is a serious omission since neglect of low-energy collective excitations and the corresponding resonances they produce can alter cross sections at low energies substantially. Thus the usual theoretical S -factor extrapolation from the quantities determined by measurement, to the energy region of astrophysical interest can be incorrect.

Recently a multi-channel algebraic scattering (MCAS) approach [17] has been developed in which the interaction of a nucleon with a nucleus is described within the collective model [18]. Extension of this approach to the treatment of the two-cluster systems has been made [19]. The MCAS model exploits the so-called orthogonalizing pseudo-potential (OPP) method [20] reformulated to take account of the influence of the Pauli exclusion principle in the collective model. This method is closely related to the OCM [8], in which the physical two-body states are obtained by renormalization that ensures their orthogonality to a certain number of unphysical bound states. The OPP method does not involve intrinsic degrees of freedom for the two fragments. Instead, an extra nonlocal contribution to the original two-body potential is constructed [17]. The MCAS approach solves the low energy coupled equations for the interacting systems, and to use information gleaned from such in capture reaction studies, the photon-emission operator has to be developed within a two-cluster coupled-channel model that also takes into account cluster sizes and the explicit coupling to their intrinsic excitations.

To construct such an operator, we start from the extension [21] of the Siegert theorem [22], expressing the electromagnetic (EM) current operator in terms of so-called generalised electric and magnetic moments of the nuclear system. Then, the amplitude of the EM transition is determined by matrix elements of that operator between nuclear states and the electric and magnetic field strengths. This explicitly gauge independent representation for the amplitude guarantees the fulfilment of the Siegert theorem for electric transitions in the long-wavelength limit. Decomposition of the EM operator into electric and magnetic parts is accomplished in a 3-dimensional form which assists in derivation of expressions for the low-energy operator of photon emission (or absorption) by a system of two extended clusters in terms of their macroscopic properties (the mean square radii, magnetic dipole moments and electric quadrupole moments) starting from the microscopic (single-particle) treatment of the total system.

Herein we present expressions for the two-cluster EM operator and for calculation of observables for low-energy nuclear radiative capture instigated by this operator. The wave functions for the initial (scattering) and final (bound) states will be built within the MCAS scheme. In the next section, the manifestly gauge independent expression for the

photon-emission amplitude based on the extension of the Siegert theorem is described. Sec. 3 contains derivation of the operator of single-photon emission by a system of two extended clusters, while the wave function construction according to the MCAS method is detailed in Sec. 4. Then, in Sec. 5, results of application of this general formalism to describe the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction are reported. Results for the cross section and the S factor are discussed and compared with experimental data, and notably that from the LUNA experiment [3]. In the context of applications of the MCAS approach, the reaction studied is considered as a simplified test calculation since it does not yet involve inelastic channels. Our future plans are to use this method to study more complex reactions where inclusion of low-lying inelastic channels will be an important aspect of the dynamics. Concluding remarks are given in Sec. 6.

2 Photon-emission amplitude based on extension of the Siegert theorem

The amplitude of transition of a nuclear system from an initial state $|\mathbf{P}_i; i\rangle$ to a state $|\mathbf{P}_f; f\rangle$ with emission of a single photon with the energy (momentum) E_γ (\mathbf{k}) and the polarization vector $\varepsilon_\nu(\mathbf{k}) \equiv (\varepsilon_0(\mathbf{k}), \boldsymbol{\varepsilon}(\mathbf{k}))$ is given by

$$\mathcal{T}_{if} = (2\pi)^4 \delta(\mathbf{P}_i - \mathbf{P}_f - \mathbf{k}) \delta(E_i - E_f - E_\gamma) T_{if}, \quad (1)$$

$$T_{if} = -\frac{1}{(2\pi)^{3/2} \sqrt{2E_\gamma}} \langle \mathbf{P}_f; f | \varepsilon^\nu J_\nu(0) | \mathbf{P}_i; i \rangle, \quad (2)$$

($\nu = 0, 1, 2, 3$)

where $J_\nu(0)$ is the EM current operator at the space-time point $x = (t, \mathbf{x}) = 0$, E_i and \mathbf{P}_i (E_f and \mathbf{P}_f) denote the total energy and momentum for the initial (final) state, and indices i and f refer to system “intrinsic” degrees of freedom.

For the operator $J_\nu(\mathbf{x})$ and the Hamiltonian H of the nuclear system, the continuity equation

$$\text{div } \mathbf{J}(\mathbf{x}) = i[H, J_0(\mathbf{x})], \quad (3)$$

has to hold. This gives rise to the gauge independence (GI) condition

$$\mathbf{k} \langle \mathbf{P}_f; f | \mathbf{J}(0) | \mathbf{P}_i; i \rangle = (E_i - E_f) \langle \mathbf{P}_f; f | \rho(0) | \mathbf{P}_i; i \rangle, \quad (4)$$

where $\rho(0) \equiv J_0(0)$, for the transition matrix element.

In practical calculations, however, this condition is often violated because of shortcomings in the description of the nuclear dynamics. For instance, for a system consisting of A_{tot} nucleons, typically the EM current is given by the one-body contribution [23],

$$\rho(0) = \rho^{[1]} \equiv \sum_{\xi=1}^{A_{\text{tot}}} \hat{e}_\xi \delta(\mathbf{r}_\xi), \quad (5)$$

$$\mathbf{J}(0) = \mathbf{J}^{[1]} \equiv \sum_{\xi=1}^{A_{\text{tot}}} \frac{\hat{e}_\xi}{2m_\xi} \{\mathbf{p}_\xi, \delta(\mathbf{r}_\xi)\} + i \sum_{\xi=1}^{A_{\text{tot}}} e \frac{\hat{\boldsymbol{\mu}}_\xi}{m_\xi} [\mathbf{s}_\xi \times \mathbf{p}_\xi, \delta(\mathbf{r}_\xi)], \quad (6)$$

where $\hat{e}_\xi = e$, $\hat{\mu}_\xi = \mu_p$ ($\hat{e}_\xi = 0$, $\hat{\mu}_\xi = \mu_n$) for protons (neutrons), e is the elementary charge, μ_p (μ_n) denotes the proton (neutron) magnetic moment in nuclear magnetons, and \mathbf{p}_ξ , \mathbf{r}_ξ and \mathbf{s}_ξ are the momentum, coordinate and spin operators for a nucleon number ξ . This current does not obey Eq. (3), and one instead has

$$[\mathbf{P}_{\text{tot}}, \mathbf{J}(0)] = [K, \rho(0)] = [H, \rho(0)] - [V, \rho(0)] , \quad (7)$$

$$\mathbf{P}_{\text{tot}} = \sum_{\xi=1}^{A_{\text{tot}}} \mathbf{p}_\xi ,$$

where K is the nuclear system kinetic-energy operator and V denotes the nuclear potential. Evidently, the GI condition, Eq. (4), is not met for the model specified by Eqs. (5) and (6) if V contains nonlocal (momentum-dependent) contributions. An interaction current has to be added to restore the gauge independence.

Furthermore, it may happen that nuclear wave functions used in calculations are not eigenstates of H corresponding to eigenvalues E_i and E_f for the initial and final state. That is the case in calculation of transition amplitudes for direct radiative capture when asymptotic radial wave functions are used for the initial and/or final states. The same problem exists in any situation where, due to the increased complexity of the boundary conditions, the scattering process is treated with a Hamiltonian that has been simplified with respect to that used for the nuclear bound state.

In view of the difficulties, which are encountered in description of the nuclear system, the representation of the amplitude of an EM process discussed in Refs. [21, 24, 25, 26] can be useful. This representation expresses T_{if} in a manifestly gauge-independent way in terms of the electric ($\mathbf{E}(\mathbf{k})$) and magnetic ($\mathbf{H}(\mathbf{k})$) field strengths, *viz.*

$$T_{if} = \mathbf{E}(\mathbf{k}) \cdot \mathbf{D}_{if}(\mathbf{k}) + \mathbf{H}(\mathbf{k}) \cdot \mathbf{M}_{if}(\mathbf{k}) , \quad (8)$$

$$\mathbf{E}(\mathbf{k}) = -i[2(2\pi)^3 E_\gamma]^{-1/2} ((E_i - E_f)\boldsymbol{\varepsilon}(\mathbf{k}) - \mathbf{k}\varepsilon_0(\mathbf{k})) , \quad (9)$$

$$\mathbf{H}(\mathbf{k}) = -i[2(2\pi)^3 E_\gamma]^{-1/2} \mathbf{k} \times \boldsymbol{\varepsilon}(\mathbf{k}) , \quad (10)$$

where $\mathbf{D}_{if}(\mathbf{k})$ and $\mathbf{M}_{if}(\mathbf{k})$ are matrix elements of the so-called generalised electric and magnetic dipole moments of the system determined by the original current J_ν . This relationship provides an extension of the Siegert theorem [22] and is an alternative to Eq. (2). According to the representation, derivation of which relies upon Eq. (3) and the diagonality of the total charge operator, we can redefine the current matrix element as

$$\langle \mathbf{P}_f; f | \rho(0) | \mathbf{P}_i; i \rangle = -i\mathbf{k} \cdot \mathbf{D}_{if}(\mathbf{k}) , \quad (11)$$

$$\langle \mathbf{P}_f; f | \mathbf{J}(0) | \mathbf{P}_i; i \rangle = -i(E_i - E_f) \mathbf{D}_{if}(\mathbf{k}) - i\mathbf{M}_{if}(\mathbf{k}) \times \mathbf{k} . \quad (12)$$

This provides the automatic fulfilment of the GI condition, Eq. (4).

Decomposition of the EM current operator into the electric and magnetic parts, which gives rise to Eq. (8), has been discussed in detail in Ref. [21]. Thus we give only a brief outline of the corresponding derivation and clarify the meaning of the quantities $\mathbf{D}_{if}(\mathbf{k})$ and $\mathbf{M}_{if}(\mathbf{k})$ by discussing their behaviour in the long-wavelength limit. In particular, we establish the link between the electric contribution in Eq. (8) to the well-known Siegert result [22].

So, we factor out the center-of-mass (c.m.) motion of the nuclear states, i.e. we use the separation

$$| \mathbf{P}_i; i \rangle = | \mathbf{P}_i \rangle | i \rangle , \quad | \mathbf{P}_i - \mathbf{k}; f \rangle = | \mathbf{P}_i - \mathbf{k} \rangle | f \rangle , \quad (13)$$

which, of course, is always valid at non-relativistic energies. Then, the translation properties of the total momentum eigenstates allow us to write

$$(\mathbf{P}_i - \mathbf{k} | = (\mathbf{P}_i | e^{i\mathbf{k}\mathbf{R}_{\text{tot}}} , \quad (14)$$

where \mathbf{R}_{tot} is the c.m. coordinate operator. The product of the exponent in Eq. (14) and the vector $\boldsymbol{\varepsilon}(\mathbf{k})$ can be decomposed into the “electric” and “magnetic” parts using the identity (cf. Ref. [24])

$$\boldsymbol{\varepsilon} e^{i\mathbf{k}\mathbf{R}_{\text{tot}}} = \int_0^1 \{ [\mathbf{P}_{\text{tot}}, i\boldsymbol{\varepsilon}\mathbf{R}_{\text{tot}} e^{i\lambda\mathbf{k}\mathbf{R}_{\text{tot}}}] + i\lambda\mathbf{R}_{\text{tot}} \times [\boldsymbol{\varepsilon} \times \mathbf{k}] e^{i\lambda\mathbf{k}\mathbf{R}_{\text{tot}}} \} d\lambda , \quad (15)$$

where the components of vectors \mathbf{P}_{tot} and \mathbf{R}_{tot} obey the usual commutation rule

$$[P_{\text{tot},j}, R_{\text{tot},l}] = -i\delta_{jl} , \quad (j, l = 1, 2, 3) .$$

Analogously,

$$\varepsilon_0 e^{i\mathbf{k}\mathbf{R}_{\text{tot}}} = \varepsilon_0 \left(1 + i\mathbf{k}\mathbf{R}_{\text{tot}} \int_0^1 e^{i\lambda\mathbf{k}\mathbf{R}_{\text{tot}}} d\lambda \right) . \quad (16)$$

Taking into account the continuity equation, Eq. (3), and as the total charge operator is diagonal, from Eqs. (2), (15) and (16) we deduce Eq. (8) with

$$\mathbf{D}_{if}(\mathbf{k}) = (2\pi)^{-3} \langle f | \mathbf{D}(\mathbf{k}) | i \rangle , \quad (17)$$

$$\mathbf{M}_{if}(\mathbf{k}) = (2\pi)^{-3} \langle f | \mathbf{M}(\mathbf{k}) | i \rangle . \quad (18)$$

The operators

$$\mathbf{D}(\mathbf{k}) = -\frac{(2\pi)^3}{E_i - E_f} \int_0^1 (\mathbf{P}_i - \lambda\mathbf{k} | \mathbf{R}_{\text{tot}} [\rho(0), H] | \mathbf{P}_i) d\lambda , \quad (19)$$

$$\mathbf{M}(\mathbf{k}) = -(2\pi)^3 \int_0^1 (\mathbf{P}_i - \lambda\mathbf{k} | \mathbf{R}_{\text{tot}} \times \mathbf{J}(0) | \mathbf{P}_i) \lambda d\lambda \quad (20)$$

act in the space of the intrinsic variables of the nuclear states. It should be stressed that Eq. (8) is quite general and can be used to describe various EM processes with nuclei at low and intermediate energies. In case of the inverse reaction (absorption of the photon with momentum \mathbf{k}), one has to replace $\lambda\mathbf{k}$ by $-\lambda\mathbf{k}$ in Eqs. (19) and (20). The results given by Eqs. (2) and (8) are identical if the GI condition of Eq. (4) holds, but can be different otherwise. We choose the representation of the amplitude given by Eq. (8) since it provides automatically the GI of calculations and helps to reproduce predictions of the low-energy theorems based on the gauge invariance. Furthermore, as will be shown in Sec. 3, this form of the amplitude is a convenient starting point for construction of effective EM interactions for a system of extended nuclear objects (clusters) with an account for their internal structure.

The representation through Eq. (8) was successfully employed in our studies [27] of pion photo- and electroproduction off nucleons and light nuclei. The current paper addresses construction of EM operators for applications at very low energies (such as nuclear radiative-capture reactions at energies of astrophysical interest).

Operators defined by Eqs. (19) and (20) act in the space of states characterised by intrinsic coordinates

$$\mathbf{r}'_{\xi} = \mathbf{r}_{\xi} - \mathbf{R}_{\text{tot}} \quad (21)$$

and momenta

$$\mathbf{p}'_{\xi} = \mathbf{p}_{\xi} - \frac{m_{\xi}}{M} \mathbf{P}_{\text{tot}} , \quad (22)$$

$$M = \sum_{\xi=1}^{A_{\text{tot}}} m_{\xi} ,$$

with commutation relations

$$[\mathbf{r}'_{\xi,j}, \mathbf{p}'_{\kappa,l}] = i(\delta_{\xi\kappa} - \frac{m_{\kappa}}{M}) \delta_{jl} \quad (23)$$

$$(\xi, \kappa = 1, 2, \dots, A_{\text{tot}} ; \quad j, l = 1, 2, 3) .$$

To clarify the meaning of the quantities specified in Eqs. (19) and (20), consider the current model as defined by Eqs. (5) and (6). After substitution of Eqs. (5) and (6) into Eqs. (19) and (20) and taking into account relations given in Eqs. (21) – (23), we obtain

$$\begin{aligned} \mathbf{D}(\mathbf{k}) = & \frac{1}{E_i - E_f} \int_0^1 d\lambda \{ [\mathbf{D}_{\text{int}}(\lambda \mathbf{k}), H_{\text{int}}] \\ & + \frac{\lambda \mathbf{k} (2\mathbf{P}_i - \lambda \mathbf{k})}{2M} \mathbf{D}_{\text{int}}(\lambda \mathbf{k}) + i \frac{\mathbf{P}_i - \lambda \mathbf{k}}{M} \rho_{\text{int}}(\lambda \mathbf{k}) \} , \end{aligned} \quad (24)$$

$$\rho_{\text{int}}(\lambda \mathbf{k}) = \sum_{\xi=1}^{A_{\text{tot}}} \hat{e}_{\xi} e^{-i\lambda \mathbf{k} \mathbf{r}'_{\xi}} , \quad (25)$$

$$\mathbf{D}_{\text{int}}(\lambda \mathbf{k}) = \sum_{\xi=1}^{A_{\text{tot}}} \hat{e}_{\xi} \mathbf{r}'_{\xi} e^{-i\lambda \mathbf{k} \mathbf{r}'_{\xi}} \quad (26)$$

for the electric generalised dipole operator and

$$\mathbf{M}(\mathbf{k}) = \mathbf{M}^{\text{orb}}(\mathbf{k}) + \mathbf{M}^{\text{spin}}(\mathbf{k}) , \quad (27)$$

$$\mathbf{M}^{\text{orb}}(\mathbf{k}) = \int_0^1 \lambda d\lambda \{ \mathbf{M}_{\text{int}}^{\text{orb}}(\lambda \mathbf{k}) - \frac{2\mathbf{P}_i - \lambda \mathbf{k}}{2M} \times \mathbf{D}_{\text{int}}(\lambda \mathbf{k}) \} , \quad (28)$$

$$\mathbf{M}^{\text{spin}}(\mathbf{k}) = \sum_{\xi=1}^{A_{\text{tot}}} e \frac{\hat{\mu}_{\xi}}{m_{\xi}} \mathbf{s}_{\xi} e^{-i\mathbf{k} \mathbf{r}'_{\xi}} + \mathbf{k} \int_0^1 \lambda^2 d\lambda \left(\sum_{\xi=1}^{A_{\text{tot}}} e \frac{\hat{\mu}_{\xi}}{m_{\xi}} (\mathbf{r}'_{\xi} \mathbf{s}_{\xi}) e^{-i\lambda \mathbf{k} \mathbf{r}'_{\xi}} \right) , \quad (29)$$

$$\mathbf{M}_{\text{int}}^{\text{orb}}(\lambda \mathbf{k}) = \sum_{\xi=1}^{A_{\text{tot}}} \frac{\hat{e}_{\xi}}{2m_{\xi}} \{ \mathbf{l}'_{\xi}, e^{-i\lambda \mathbf{k} \mathbf{r}'_{\xi}} \} , \quad (30)$$

$$\mathbf{l}'_{\xi} \equiv \mathbf{r}'_{\xi} \times \mathbf{p}'_{\xi} \quad (31)$$

for the magnetic one. Obviously, the second term in Eq. (29) does not contribute to the amplitude in Eq. (8). Moreover, for a \mathbf{k} -congruent frame ($\mathbf{P}_i \times \mathbf{k} = 0$) which will be

implied below, contributions from the second term of Eq. (24) and from the integrand in Eq. (28) cancel when emission of a real photon is considered. In that case one usually uses the Coulomb gauge,

$$\varepsilon_0(\mathbf{k}) = 0, \quad \mathbf{k}\varepsilon(\mathbf{k}) = 0, \quad (32)$$

for which the last term in Eq. (24) also vanishes. Partial cancellation between the two terms of Eq. (8) reflects the fact that, in general, decomposition of an EM transition amplitude into electric and magnetic parts is not unique (see, e.g., discussion in Ref. [25]).

Thus, as a result, we have

$$\mathbf{D}(\mathbf{k}) = \frac{E_i^{\text{int}} - E_f^{\text{int}}}{E_i - E_f} \int_0^1 \mathbf{D}_{\text{int}}(\lambda\mathbf{k}) d\lambda, \quad (33)$$

$$\mathbf{M}(\mathbf{k}) = \int_0^1 \mathbf{M}_{\text{int}}^{\text{orb}}(\lambda\mathbf{k}) \lambda d\lambda + \sum_{\xi=1}^{A_{\text{tot}}} e \frac{\hat{\mu}_{\xi}}{m_{\xi}} \mathbf{s}_{\xi} e^{-i\mathbf{k}\mathbf{r}'_{\xi}}, \quad (34)$$

where E_i^{int} (E_f^{int}) is the intrinsic energy of the initial (final) nuclear system. Calculation of the amplitude of interest is reduced to calculation of the matrix elements of operators \mathbf{D}_{int} , $\mathbf{M}_{\text{int}}^{\text{orb}}$ and \mathbf{M}^{spin} between the intrinsic states of the nuclear system. From Eq. (33) the classical Siegert result [22] for the electric transitions in the long-wavelength limit $\mathbf{k} \rightarrow 0$ is readily deduced. In this limit, the operator $\mathbf{M}(\mathbf{k})$ becomes that of the total magnetic moment for the system. Unlike the common procedure [23, 28] of obtaining the electric and magnetic multipole operators, Eq. (8) with quantities \mathbf{D}_{if} and \mathbf{M}_{if} determined by Eqs. (33) and (34) decomposes the amplitude into the electric and magnetic parts without using partial-wave expansion, and no approximations valid only for low photon energies are made. The operators given in Eqs. (33) and (34) are three-dimensional involving an integration over λ that can be easily carried out. In the next section we show that this representation in terms of generalised electric and magnetic dipole operators, is of particular use in derivation of photon-emission operators for cluster-type nuclear structures. In fact, with Eqs. (33) and (34), general expressions for the operator of photon emission by a system of two (or more) finite-size composite objects can be derived while taking their internal cluster structure into account.

3 Operator of single-photon emission by a two-cluster system

Assume now that the nuclear system consists of two subsystems (clusters) A (A nucleons) and B ($B=A_{\text{tot}}-A$ nucleons) and is described by a wave function determined by subsystem intrinsic states $|\phi^A\rangle$ and $|\phi^B\rangle$ and a vector $|\Psi_{AB}\rangle$ corresponding to the di-cluster relative motion. We can introduce for such a system a new set of coordinates and momenta as follows

$$\mathbf{r}'_{\alpha} = \mathbf{r}_{\alpha}^A + \frac{M_B}{M} \mathbf{R}, \quad \mathbf{r}'_{\beta} = \mathbf{r}_{\beta}^B - \frac{M_A}{M} \mathbf{R}, \quad (35)$$

$$\mathbf{p}'_{\alpha} = \mathbf{p}_{\alpha}^A + \frac{m_{\alpha}}{M_A} \mathbf{P}, \quad \mathbf{p}'_{\beta} = \mathbf{p}_{\beta}^B - \frac{m_{\beta}}{M_B} \mathbf{P}, \quad (36)$$

$$\mathbf{R} = \mathbf{R}_A - \mathbf{R}_B, \quad \mathbf{P} = \frac{M_B \mathbf{P}_A - M_A \mathbf{P}_B}{M}, \quad (37)$$

$$(\alpha = 1, 2, \dots, A; \quad \beta = 1, 2, \dots, B)$$

where $\mathbf{R}_{A,B}$ ($\mathbf{P}_{A,B}$) and $M_{A,B}$ are the coordinate (momentum) and mass of the subsystem A, B; and \mathbf{r}_α^A , \mathbf{r}_β^B (\mathbf{p}_α^A , \mathbf{p}_β^B) refers to the “intrinsic” coordinate (momentum) of a nucleon, which belongs to cluster A, B.

For the two-cluster relative coordinate \mathbf{R} and momentum \mathbf{P} and the subsystem intrinsic operators, the following commutation relations hold

$$\begin{aligned} [R_j, P_l] &= i\delta_{jl} , \quad [r_{\xi,j}^{A,B}, p_{\kappa,l}^{A,B}] = i(\delta_{\xi\kappa} - \frac{m_\kappa}{M_{A,B}}) \delta_{jl} , \\ [r_{\xi,j}^{A,B}, p_{\kappa,l}^{B,A}] &= [r_{\xi,j}^{A,B}, P_l] = [R_j, p_{\kappa,l}^{A,B}] = 0 , \end{aligned} \quad (38)$$

$$(\xi, \kappa = 1, 2, \dots, A [B] \text{ for cluster A [B]; } j, l = 1, 2, 3) .$$

They can be obtained straightforwardly from Eqs. (35) – (37) with the help of Eq. (23) and manifest the reciprocal independence of the intrinsic coordinates and momenta for the two different subsystems from each other.

Then, substitution of Eqs. (35) and (36) into Eq. (26) gives

$$\mathbf{D}_{\text{int}}(\lambda\mathbf{k}) = \mathbf{d}_A(\lambda\mathbf{k}) e^{-i\lambda\frac{M_B}{M}\mathbf{k}\mathbf{R}} + \mathbf{d}_B(\lambda\mathbf{k}) e^{i\lambda\frac{M_A}{M}\mathbf{k}\mathbf{R}} + \mathbf{D}_{AB}(\lambda\mathbf{k}) , \quad (39)$$

$$\mathbf{d}_{A,B}(\lambda\mathbf{k}) = \sum_{\xi=1}^{A,B} \hat{e}_\xi \mathbf{r}_\xi^{A,B} e^{-i\lambda\mathbf{k}\mathbf{r}_\xi^{A,B}} , \quad (40)$$

$$\mathbf{D}_{AB}(\lambda\mathbf{k}) = \frac{M_B}{M} \rho_A(\lambda\mathbf{k}) \mathbf{R} e^{-i\lambda\frac{M_B}{M}\mathbf{k}\mathbf{R}} - \frac{M_A}{M} \rho_B(\lambda\mathbf{k}) \mathbf{R} e^{i\lambda\frac{M_A}{M}\mathbf{k}\mathbf{R}} , \quad (41)$$

$$\rho_{A,B}(\lambda\mathbf{k}) = \sum_{\xi=1}^{A,B} \hat{e}_\xi e^{-i\lambda\mathbf{k}\mathbf{r}_\xi^{A,B}} . \quad (42)$$

It should be stressed that Eq. (33) for the generalised electric dipole operator, with $\mathbf{D}_{\text{int}}(\lambda\mathbf{k})$ determined by Eqs. (39) – (42), is valid to exactly the same extent as is the one-body current of Eqs. (5) and (6). Eq. (39) provides a convenient starting point to construct electric transition operators of different form depending on a particular situation. Consider, for instance, low-energy photon emission by a system of two charged clusters with substantially different sizes. In addition, due to a strong Coulomb repulsion, on average the emission (absorption) process occurs at a very large separation distance, considerably greater than the characteristic size of the bigger cluster. Then, one can make a low-energy expansion of Eq. (39) with respect to specific dynamical variables, that is independent of the remaining variables. In other words, it is possible to obtain evaluations at different perturbative orders with respect to the three separated expansion parameters $\mathbf{k}\mathbf{r}_\alpha^A$, $\mathbf{k}\mathbf{r}_\beta^B$ and $\mathbf{k}\mathbf{R}$.

In general, the quantity $\mathbf{d}_{A,B}(\lambda\mathbf{k})$ in Eq. (39) gives rise to a generalised electric dipole operator for the subsystem A, B and contains all multipole contributions responsible for the electric transitions in this subsystem. Analogously, the operator $\mathbf{D}_{AB}(\lambda\mathbf{k})$ provides (for all orders in \mathbf{k}) the electric transition contribution due to the relative motion of the two clusters. But, as follows from Eq. (39), the separation between the “intrinsic” (for the subsystems A and B) transitions and the ones due to the relative di-cluster motion is not complete. The “intrinsic” electric operators $\mathbf{d}_A(\lambda\mathbf{k})$ and $\mathbf{d}_B(\lambda\mathbf{k})$ in this formula are

coupled to the exponential factors dependent on the two-cluster relative coordinate, while the quantity $\mathbf{D}_{AB}(\lambda\mathbf{k})$ includes cluster form-factor operators $\rho_A(\lambda\mathbf{k})$ and $\rho_B(\lambda\mathbf{k})$.

However, such a separation is exact in the long-wavelength limit in which, from Eqs. (33) and (39), one gets

$$\mathbf{D}(0) = \mathbf{d}^A + \mathbf{d}^B + \mathbf{D}^{AB}, \quad (43)$$

$$\mathbf{D}^{AB} = eC_{E1}\mathbf{R}, \quad (44)$$

$$C_{E1} = \frac{M_B Z_A - M_A Z_B}{M}, \quad (45)$$

where $\mathbf{d}^{A,B} \equiv \mathbf{d}_{A,B}(0)$ is the usual electric dipole operator of the system A, B [23, 28]; Z_A (Z_B) is the component A (B) charge, and \mathbf{D}^{AB} is the electric dipole operator for two point-like clusters. This proves that the constructed electric operator fulfils the Siegert theorem [22] at $\mathbf{k} \rightarrow 0$. Note also that there is no recoil in this limit. Therefore the energy factor in Eq. (33) is equal to 1.

If the states $|\chi_A\rangle$ and $|\chi_B\rangle$ possess a definite parity, and the intrinsic parities of the clusters do not change in the EM transitions, by neglecting terms of order $O(k^2)$ in Eq. (39) we obtain

$$\mathbf{D}(\mathbf{k}) = \frac{E_i^{\text{int}} - E_f^{\text{int}}}{E_i - E_f} \left(\mathbf{D}^{E1} - \frac{i}{6} \mathbf{D}^{E2}(\mathbf{k}) \right), \quad (46)$$

$$\mathbf{D}^{E1} \equiv \mathbf{D}^{AB}, \quad (47)$$

$$D_j^{E2}(\mathbf{k}) = \sum_l k_l \left(t_{jl}^A + t_{jl}^B + eC_{E2} T_{jl}^{AB} \right), \quad (48)$$

$$t_{jl}^{A,B} = \sum_{\xi=1}^{A,B} \hat{e}_\xi \left(3 r_{\xi,j}^{A,B} r_{\xi,l}^{A,B} - \delta_{jl} [r_\xi^{A,B}]^2 \right), \quad (49)$$

$$T_{jl}^{AB} = 3R_j R_l - \delta_{jl} R^2, \quad (50)$$

$$C_{E2} = \frac{M_B^2 Z_A + M_A^2 Z_B}{M^2}, \quad (51)$$

$$(j, l = 1, 2, 3).$$

Here, we have taken into account the gauge condition, Eq. (32). The operator in the parentheses of Eq. (48) is the electric quadrupole tensor operator for the two-cluster system.

Of course, one can proceed with analysis of Eq. (39) and obtain higher-order contributions to the electric operator defined by Eq. (33). In particular, a correction to the two-cluster electric dipole operator \mathbf{D}^{AB} emerges due to the finite cluster sizes. This correction can be of great importance when the radiative capture occurs with different symmetric nuclei, namely, those having equal numbers of protons and neutrons. For those cases, the quantity given by Eq. (45) is vanishingly small. Examples are the radiative capture of ^4He by ^{12}C , by ^{16}O , or by some heavier nuclei. That capture process is very important in assessing stellar evolution and nucleo-synthesis. For simplicity, assume that the reaction is analysed within a potential cluster model with no coupling to intrinsic excitations of the subsystems A and B (*i.e.*, they are considered as “frozen”), and that the

cluster electric quadrupole moments are zero. Then, for sufficiently small photon energy ($\mathbf{k}r_\xi^{A,B} \ll 1$), from Eq. (39) the correction

$$C_{E1} \rightarrow C_{E1}(k) \equiv C_{E1} + k^2 \Delta C_{E1} , \quad (52)$$

$$\Delta C_{E1} = -\frac{1}{18} \frac{M_B Z_A \langle r_A^2 \rangle - M_A Z_B \langle r_B^2 \rangle}{M_A + M_B} , \quad (53)$$

is found. Therein $\langle r_{A,B}^2 \rangle$ is the mean square charge radius of the cluster A, B. However, the corresponding contribution to the operator $\mathbf{D}(\mathbf{k})$ has to be taken into account together with other terms in Eq. (33) of the same order in \mathbf{k} .

The same reasoning can be also applied to the magnetic operator given in Eq. (34). Substituting Eqs. (35) and (36) into Eq. (30) gives

$$\begin{aligned} \mathbf{M}_{\text{int}}^{\text{orb}}(\lambda \mathbf{k}) &= \mathbf{m}_A^{\text{orb}}(\lambda \mathbf{k}) e^{-i\lambda \frac{M_B}{M} \mathbf{k} \mathbf{R}} + \mathbf{m}_B^{\text{orb}}(\lambda \mathbf{k}) e^{i\lambda \frac{M_A}{M} \mathbf{k} \mathbf{R}} + \mathbf{M}_{AB}^{\text{orb}}(\lambda \mathbf{k}) \\ &+ \frac{M_B}{2M} \{ \mathbf{d}_A(\lambda \mathbf{k}) \times \mathbf{V}, e^{-i\lambda \frac{M_B}{M} \mathbf{k} \mathbf{R}} \} - \frac{M_A}{2M} \{ \mathbf{d}_B(\lambda \mathbf{k}) \times \mathbf{V}, e^{i\lambda \frac{M_A}{M} \mathbf{k} \mathbf{R}} \} \\ &+ \frac{M_B}{M} [\mathbf{R} \times \mathbf{v}_A(\lambda \mathbf{k})] e^{-i\lambda \frac{M_B}{M} \mathbf{k} \mathbf{R}} - \frac{M_A}{M} [\mathbf{R} \times \mathbf{v}_B(\lambda \mathbf{k})] e^{i\lambda \frac{M_A}{M} \mathbf{k} \mathbf{R}} , \end{aligned} \quad (54)$$

$$\mathbf{m}_{A,B}^{\text{orb}}(\lambda \mathbf{k}) = \sum_{\xi=1}^{A,B} \frac{\hat{e}_\xi}{2m_\xi} \{ \mathbf{l}_\xi^{A,B}, e^{-i\lambda \mathbf{k} \mathbf{r}_\xi^{A,B}} \} , \quad (55)$$

$$\mathbf{M}_{AB}^{\text{orb}}(\lambda \mathbf{k}) = \frac{1}{2M} \frac{M_B}{M_A} \rho_A(\lambda \mathbf{k}) \{ \mathbf{L}, e^{-i\lambda \frac{M_B}{M} \mathbf{k} \mathbf{R}} \} + \frac{1}{2M} \frac{M_A}{M_B} \rho_B(\lambda \mathbf{k}) \{ \mathbf{L}, e^{i\lambda \frac{M_A}{M} \mathbf{k} \mathbf{R}} \} , \quad (56)$$

$$\mathbf{v}_{A,B}(\lambda \mathbf{k}) = \frac{1}{2} \sum_{\xi=1}^{A,B} \hat{e}_\xi \{ \mathbf{v}_\xi^{A,B}, e^{-i\lambda \mathbf{k} \mathbf{r}_\xi^{A,B}} \} , \quad (57)$$

$$\mathbf{v}_\xi^{A,B} = \frac{\mathbf{p}_\xi^{A,B}}{m_\xi} , \quad \mathbf{V} = \frac{\mathbf{P}}{M_{AB}} , \quad M_{AB} = \frac{M_A M_B}{M} , \quad (58)$$

$$\mathbf{l}_\xi^{A,B} = \mathbf{r}_\xi^{A,B} \times \mathbf{p}_\xi^{A,B} , \quad \mathbf{L} = \mathbf{R} \times \mathbf{P} . \quad (59)$$

The structure of the first line in Eq. (54) is very similar to the structure of Eq. (39) for the electric transition operator. However, the magnetic operator couples the cluster “intrinsic” EM transitions to the relative motion between clusters in a more complicated way than does the corresponding electric operator. That is so because it also contains vector products of the subsystem intrinsic operators $\mathbf{d}_{A,B}$ and $\mathbf{v}_{A,B}$ with the two-cluster relative momentum and coordinate.

Similarly, the magnetic spin operator of Eq. (29) cast in terms of the two-cluster coordinates defined by Eq. (35) becomes

$$\mathbf{M}^{\text{spin}}(\mathbf{k}) = \mathbf{m}_A^{\text{spin}}(\mathbf{k}) e^{-i\lambda \frac{M_B}{M} \mathbf{k} \mathbf{R}} + \mathbf{m}_B^{\text{spin}}(\mathbf{k}) e^{i\lambda \frac{M_A}{M} \mathbf{k} \mathbf{R}} , \quad (60)$$

$$\mathbf{m}_{A,B}^{\text{spin}}(\mathbf{k}) = \sum_{\xi=1}^{A,B} e \frac{\hat{\mu}_\xi}{m_\xi} \mathbf{s}_\xi e^{-i\lambda \mathbf{k} \mathbf{r}_\xi^{A,B}} , \quad (61)$$

where the Coulomb gauge, given by Eq. (32), has been assumed.

As was done in the analysis of the operator $\mathbf{D}(\mathbf{k})$, we assume that the magnetic operator defined in Eq. (34) acts in a space of intrinsic states possessing a definite parity. Also consider the part of this operator which gives rise to transitions that do not change parity. Then, in the limit $\mathbf{k} \rightarrow 0$, all but the first three terms in Eq. (54) can be omitted and we can write

$$\mathbf{M}(0) = \boldsymbol{\mu}^A + \boldsymbol{\mu}^B + \mathbf{M}^{AB} , \quad (62)$$

$$\boldsymbol{\mu}^{A,B} = \sum_{\xi=1}^{A,B} \frac{1}{m_{\xi}} \left(\frac{\hat{e}_{\xi}}{2} \mathbf{l}_{\xi}^{A,B} + e \hat{\mu}_{\xi} \mathbf{s}_{\xi} \right) , \quad (63)$$

$$\mathbf{M}^{AB} = \mu_N C_{M1} \mathbf{L} , \quad (64)$$

$$C_{M1} = \frac{1}{A_{\text{tot}}} \left(\frac{M_B}{M_A} Z_A + \frac{M_A}{M_B} Z_B \right) , \quad (65)$$

where $\mu_N \equiv e/2m_N$ is the nuclear magneton and m_N is the nucleon mass. Therefore, the generalised magnetic dipole moment determined by Eqs. (54) and (60) in the long-wavelength limit is the total magnetic dipole moment operator for the two-body system. This is a sum of the subsystem intrinsic magnetic dipole operators $\boldsymbol{\mu}^{A,B}$ defined in the usual way [23, 28] with an additional contribution \mathbf{M}^{AB} that is due to cluster-relative orbital motion. Evidently, in the specific case of a two-cluster potential picture without coupling to “intrinsic” excitations for the subsystems, transition matrix elements of operators $\boldsymbol{\mu}^{A,B}$ are determined by the static magnetic dipole moments of the clusters A and B.

We can proceed with the analysis of the magnetic operator given in Eq. (34) for low photon energies in a similar manner to that we have used for the electric operator. Disregarding terms of order $O(\mu_N k^2)$, the part of this operator which does not change the cluster intrinsic parity can be written as

$$\mathbf{M}(\mathbf{k}) = \mathbf{M}^{M1} - i \mathbf{M}^{M2}(\mathbf{k}) , \quad (66)$$

$$\mathbf{M}^{M1} \equiv \mathbf{M}(0) , \quad (67)$$

$$\begin{aligned} \mathbf{M}^{M2}(\mathbf{k}) = & (\mathbf{kR}) \left(\frac{M_B}{M} \boldsymbol{\mu}^A - \frac{M_A}{M} \boldsymbol{\mu}^B \right) + \mu_N C_{M2} \{ \mathbf{L}, (\mathbf{kR}) \} \\ & + \frac{i}{6} \mathbf{R} \times \left(\frac{M_B}{M} [H_A, \mathbf{d}_A^{E2}(\mathbf{k})] - \frac{M_A}{M} [H_B, \mathbf{d}_B^{E2}(\mathbf{k})] \right) \\ & + \frac{1}{3} \left(\frac{M_B}{M} \mathbf{d}_A^{E2}(\mathbf{k}) - \frac{M_A}{M} \mathbf{d}_B^{E2}(\mathbf{k}) \right) \times \mathbf{V} , \end{aligned} \quad (68)$$

$$d_{A,B; j}^{E2}(\mathbf{k}) = \frac{1}{3} \left(k_i t_{ji}^{A,B} + e k_j Z_{A,B} r_{A,B}^2 \right) , \quad (69)$$

$$C_{M2} = \frac{1}{3A_{\text{tot}}M} \left(\frac{M_B^2}{M_A} Z_A - \frac{M_A^2}{M_B} Z_B \right) , \quad (70)$$

where H_A (H_B) and r_A^2 (r_B^2) are the cluster A (B) intrinsic Hamiltonian and the operator of the square charge radius, respectively. Details are given in Appendix A. If we consider

the case of a cluster-like potential model, with no coupling to the intrinsic excitations and with vanishing static electric quadrupole moments, Eq. (68) reduces to

$$\begin{aligned} \mathbf{M}^{M2}(\mathbf{k}) = & (\mathbf{k}\mathbf{R}) \left(\frac{M_B}{M} \boldsymbol{\mu}^A - \frac{M_A}{M} \boldsymbol{\mu}^B \right) + \mu_N C_{M2} \{ \mathbf{L}, (\mathbf{k}\mathbf{R}) \} \\ & + \frac{1}{9} e \frac{M_B Z_A \langle r_A^2 \rangle - M_A Z_B \langle r_B^2 \rangle}{M} \mathbf{k} \times \mathbf{V} . \end{aligned} \quad (71)$$

Finally, note that the formalism outlined here paves the way for the development of an extended cluster model for description of low-energy photo-nuclear processes taking into account the internal structure and EM excitation of the clusters. Starting from the microscopic (single-particle) model defined by Eqs. (5) and (6) for the EM current operator, we have derived the low-energy EM transition operators, and specifically separated the terms acting on the part of the wave function that describes the relative motion between clusters.

Conversely, the cluster intrinsic structure is described by operators of cluster magnetic dipole and electric quadrupole moments and squares of charge radii. Usually, these intrinsic operators are defined microscopically, but the evaluation of their matrix elements can be done within any model, be it microscopic (e.g., by the shell model, or by few-body techniques for light nuclei) or macroscopic (a collective-type model). Therefore, our approach can be considered as a step towards construction of a unified framework for treatment of static electromagnetic properties of nuclei and low-energy nuclear radiative capture with a link (as shown below) to the description of nuclear two-body scattering within the multichannel approach of Ref. [17]. Furthermore, this formalism can be easily generalised to the case of three-cluster and more complex systems through the proper extension of the set of the Jacobi coordinates and momenta.

4 MCAS wave functions for two-cluster states

Consider the radiative-capture process

$$A + B \rightarrow C + \gamma . \quad (72)$$

Initially there is a scattering state $|\Psi_{\mathbf{P}}^{(+)}; \phi_{J_A}^A J_A m_A; \phi_{J_B}^B J_B m_B\rangle$ characterised by the two-body c.m. momentum \mathbf{P} and a nucleus A (B) intrinsic state $\phi_{J_A}^A$ ($\phi_{J_B}^B$) with the total spin J_A (J_B) and its projection m_A (m_B). The process forms a bound state $|\Phi_{J_C}^C J_C m_C\rangle$ which has total spin J_C and projection m_C . Assume that the nuclear system is described within a two-cluster picture without any direct reference to single-particle coordinates and momenta. This conforms to the current MCAS approach [17], which utilises the OPP method [20, 29] to account for the Pauli principle. With partial-wave decomposition, the two-cluster initial- and final-state wave functions can be written as

$$\begin{aligned} \langle \mathbf{R} | \Psi_{\mathbf{P}}^{(+)}; \phi_{J_A}^A J_A m_A; \phi_{J_B}^B J_B m_B \rangle = & \sum_{\substack{J, a, a'' \\ m_A'', m_B''}} \langle a'', m_A'', m_B'' | \mathcal{Y}_{JM_J}(\Omega_{\mathbf{R}}, \Omega_{\mathbf{P}}) | a, m_A, m_B \rangle \\ & \times | \phi_{J_A}^A J_A'' m_A'' \rangle | \phi_{J_B}^B J_B'' m_B'' \rangle \Psi_{a'' a}^{J (+)}(R) , \end{aligned} \quad (73)$$

and

$$\begin{aligned} \langle \mathbf{R} | \Phi_{J_C}^C J_C m_C \rangle &= \sum_{\substack{a', M', M'_L, \\ m'_A, m'_B}} C(L' M'_L J'_A m'_A J' M' J'_B m'_B J_C m_C) \\ &\times | \phi_{J'_A}^A J'_A m'_A \rangle | \phi_{J'_B}^B J'_B m'_B \rangle \Phi_{J_C a'}^C(R) Y_{L' M'_L}(\Omega_{\mathbf{R}}) , \end{aligned} \quad (74)$$

wherein we have used the short notations

$$\begin{aligned} | a \rangle &\equiv | P; ((L, J_A) J''', J_B) J \rangle , \\ | a' \rangle &\equiv | P'; ((L', J'_A) J', J'_B) J_C \rangle , \\ | a'' \rangle &\equiv | P''; ((L'', J''_A) J'', J''_B) J \rangle , \end{aligned} \quad (75)$$

for the channel states. We have also made definitions

$$\begin{aligned} \langle a'', m''_A, m''_B | \mathcal{Y}_{JM_J}(\Omega_{\mathbf{R}}, \Omega_{\mathbf{P}}) | a, m_A, m_B \rangle &\equiv \sum_{\substack{M, M'', M''', \\ M_L, M''_L}} C(L'' M''_L J''_A m''_A J'' M'' J''_B m''_B J M_J) \\ &\times C(L M_L J_A m_A J''' M''' J_B m_B J M_J) Y_{L'' M''_L}(\Omega_{\mathbf{R}}) Y_{L M_L}^*(\Omega_{\mathbf{P}}) , \end{aligned} \quad (76)$$

$$C(j_1 m_1 j_2 m_2 j_3 m_3 j_4 m_4 j_5 m_5) \equiv \langle j_1 m_1 j_2 m_2 | j_3 m_3 \rangle \langle j_3 m_3 j_4 m_4 | j_5 m_5 \rangle . \quad (77)$$

These formulae are given for a general case when non-central nuclear forces [30] and in-elastic ($P'' \neq P$) two-body channels are to be taken into account. The angular-momentum coupling scheme implied is shown explicitly in Eqs. (75).

The coordinate-space scattering radial wave function $\Psi_{a''a}^{J(+)}(R)$ has an asymptotic behaviour [31]

$$\Psi_{a''a}^{J(+)}(R) \xrightarrow{R \rightarrow \infty} \sqrt{\frac{2}{\pi}} \frac{i^{L''}}{P R} \frac{1}{2i} \left(\sqrt{\frac{P}{P''}} S_{a''a}^J O_{L''}^{(+)}(P'' R) - \delta_{a''a} O_L^{(-)}(P R) \right) , \quad (78)$$

where $S_{a''a}^J$ is an element of the partial-wave S -matrix and $O_a^{(-)}$ and $O_a^{(+)}$ are the incoming and outgoing Coulomb waves respectively. For the latter, we imply [32]

$$\begin{aligned} O_L^{(-)}(P R) &\xrightarrow{R \rightarrow \infty} e^{-i[PR - \eta \ln(2PR) - \frac{L\pi}{2}]} , \\ O_L^{(+)}(P R) &\xrightarrow{R \rightarrow \infty} e^{i[PR - \eta \ln(2PR) - \frac{L\pi}{2}]} , \end{aligned} \quad (79)$$

$$\eta \equiv \eta(P) = Z_A Z_B \alpha M_{AB} / P , \quad (80)$$

with α being the fine-structure constant. Next, we can write

$$\begin{aligned} O_L^{(-)}(P R) &= O_L^{R(-)}(P R) e^{i\sigma_L(P)} , \\ O_{L''}^{(+)}(P'' R) &= O_{L''}^{R(+)}(P'' R) e^{-i\sigma_{L''}(P'')} , \end{aligned} \quad (81)$$

$$O_L^{R(\pm)}(x) = G_L(x) \pm i F_L(x) , \quad (82)$$

where, as usual, for the Coulomb regular (F_L) and irregular (G_L) functions, one has

$$\begin{aligned} F_L(P R) &\xrightarrow{R \rightarrow \infty} \sin[PR - \eta \ln(2PR) - \frac{L\pi}{2} + \sigma_L(P)] , \\ G_L(P R) &\xrightarrow{R \rightarrow \infty} \cos[PR - \eta \ln(2PR) - \frac{L\pi}{2} + \sigma_L(P)] . \end{aligned} \quad (83)$$

The Coulomb phase shift depends on the Sommerfeld parameter η as

$$\sigma_L(P) = \arg \Gamma(L + 1 + i\eta(P)) .$$

Using the definitions given in Eq. (81) and introducing the “reduced” S matrix S_f^R [30] through the definition

$$S_{a''a}^J = e^{i\sigma_{L''}(P'')} S_{J,a''a}^R e^{i\sigma_L(P)} , \quad (84)$$

the asymptotic condition (78) becomes

$$\Psi_{a''a}^{J(+)}(R) \xrightarrow{R \rightarrow \infty} i^{L''} \sqrt{\frac{2}{\pi}} \frac{e^{i\sigma_L(P)}}{\sqrt{P''P}R} \frac{S_{J,a''a}^R O_{L''}^{R(+)}(P''R) - \delta_{a''a} O_L^{R(-)}(PR)}{2i} . \quad (85)$$

The wave function $\Psi_{a''a}^{J(+)}(R)$, which is regular at $R = 0$ and satisfies the asymptotic condition, Eq. (85), can be constructed from the MCAS multi-channel scattering matrix given in the Coulomb-state representation [33] for a separable two-body potential of rank N , namely

$$V_{a''a}(p, q) = \sum_{n=1}^N \langle p | \chi_{a''n} \rangle \lambda_n^{-1} \langle \chi_{an} | q \rangle , \quad (86)$$

in which the momentum-space potential form factors are the Fourier-Coulomb transforms,

$$\hat{\chi}_{an}(p) \equiv \langle p | \chi_{an} \rangle = \sqrt{\frac{2}{\pi}} \frac{1}{p} \int_0^\infty F_L(pr) \chi_{an}(r) dr , \quad (87)$$

of the corresponding coordinate-space form factors.

In our present study, we employ the procedure [34] to construct the scattering radial wave function for the separable potential (86) in coordinate space. That involves a direct resolution of the coupled-channel Lippmann-Schwinger equation in a finite-rank matrix form using the Green’s function expressed through the Coulomb functions and for the physical outgoing solution, i.e., that obeying condition given in Eq. (85). Following this procedure with the MCAS scattering matrix [17], we get

$$\Psi_{a''a}^{J(+)}(R) = i^{L''} \sqrt{\frac{2}{\pi}} \frac{\exp[i\sigma_L(P)]}{\sqrt{P''P}R} \left[F_L(PR) \delta_{a''a} - \pi M_{AB} \sqrt{P''P} \Phi_{a''a}^{J(+)}(R) \right] , \quad (88)$$

$$\begin{aligned} \Phi_{a''a}^{J(+)}(R) = i^{L-L''} \sum_{n,n'=1}^N \left\{ F_{L''}(P''R) \chi_{a''n}^G(P'', R) - G_{L''}(P''R) \chi_{a''n}^F(P'', R) \right. \\ \left. + O_{L''}^{R(+)}(P''R) \hat{\chi}_{a''n}(P'') \right\} \left[\lambda - \mathcal{G}_0^{(+)} \right]_{nn'}^{-1} \hat{\chi}_{an'}(P) , \quad (89) \end{aligned}$$

where

$$\begin{aligned} \chi_{an}^G(P, R) &\equiv \sqrt{\frac{2}{\pi}} \frac{1}{P} \int_R^\infty G_L(Pr) \chi_{an}(r) dr , \\ \chi_{an}^F(P, R) &\equiv \sqrt{\frac{2}{\pi}} \frac{1}{P} \int_R^\infty F_L(Pr) \chi_{an}(r) dr , \\ \hat{\chi}_{an}(P) &\equiv \chi_{an}^F(P, 0) . \end{aligned} \quad (90)$$

The quantity in the square brackets of Eq. (89) is a matrix, with its elements in the space of separable-decomposition indices being

$$\begin{aligned} [\mathcal{G}_0^{(+)}]_{nn'} = 2M_{AB} & \left[\sum_{a=1}^{\text{open}} \int_0^\infty \hat{\chi}_{an}(x) \frac{x^2}{P^2 - x^2 + i0} \hat{\chi}_{an'}(x) dx \right. \\ & \left. - \sum_a^{\text{closed}} \int_0^\infty \hat{\chi}_{an}(x) \frac{x^2}{h_a^2 + x^2} \hat{\chi}_{an'}(x) dx \right], \end{aligned} \quad (91)$$

where

$$[\lambda]_{nn'} = \lambda_n \delta_{nn'}, \quad (92)$$

$$P = \sqrt{2M_{AB}(E_{\text{c.m.}} - E_a^{\text{th}})}, \quad h_a = \sqrt{2M_{AB}(E_a^{\text{th}} - E_{\text{c.m.}})}, \quad (93)$$

where E_a^{th} is the threshold energy of channel a and $E_{\text{c.m.}}$ is the elastic-channel kinetic energy. Both are considered in the c.m. frame. The function given in Eq. (89) oscillates at large radii with amplitude given by the multi-channel T matrix. Thus we can write

$$O_{a''a}^{R(+)}(\infty) T_{a''a}^J = \Phi_{a''a}^{J(+)}(\infty), \quad (94)$$

$$S_{J,a''a}^R = \delta_{a''a} - 2\pi i M_{AB} \sqrt{P''P} T_{a''a}^J, \quad (95)$$

where the asymptotic outgoing Coulomb wave $O_{a''a}^{R(+)}(\infty)$ is given by Eqs. (79) and (81).

Thus, we have constructed the radial wave function for the two-fragment initial (scattering) state, which is general in the sense that possibilities of coupling to inelastic channels are taken into consideration along with coupling between elastic channels due to the non-central part of the two-body potential. The wave function construction for bound states can be fulfilled through solving the coupled-channel Schrödinger problem analogously to the procedure outlined above for the scattering wave function. That is so as the MCAS approach utilises Sturmian expansions to construct the separable interactions, with which determination of both the positive-energy (resonance) and negative-energy (bound) two-body states solutions of the Lippmann-Schwinger equations can be found based upon analytic properties of the Sturmian functions. As all details have been given [17], we do not reproduce them here. We conclude this section by noting that the inversion of the matrix $[\lambda - \mathcal{G}_0^{(+)}]$ contains all the dynamical features of the two-cluster interaction process, including possible formation of a compound system at the given resonant energy.

5 Reaction ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$

The formalism outlined above encompasses studies of EM transitions in which the role of the finite sizes and collective excitations of the fragments can be taken into account. However, for an initial application of the formalism, we have chosen the reaction ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$. In this case there are no low-energy inelastic two-body channels, whereas the elastic channels are uncoupled. Such a choice has been governed in part by the amount of experimental information available as well as the many theoretical studies made of it in the past. Furthermore, a goal of this illustrative calculation was to establish the degree of applicability of the MCAS model in a description of EM processes. We also wish to compare theoretical ingredients of this calculation with those given by other approaches. In the first instance we have not sought to achieve a quantitative agreement with experimental data. Thus

we have not varied parameters of the two-body potential in our calculation. Instead, the radial wave function's for the initial and final states were constructed using the nuclear potential of Ref. [19] which had been found to best reproduce the states and resonance widths of ${}^7\text{Li}$.

The differential cross section for the radiative-capture process can be written as

$$\frac{d\sigma^{\text{c.m.}}}{d\Omega_\gamma} = 4\pi^2\alpha \frac{M_{\text{AB}}}{|\mathbf{P}|} \frac{M}{M + E_\gamma} E_\gamma^3 \left(|\widetilde{\mathbf{D}}_{if}|^2 - |\widehat{\mathbf{k}}\widetilde{\mathbf{D}}_{if}|^2 + |\widetilde{\mathbf{M}}_{if} \times \widehat{\mathbf{k}}|^2 + 2\text{Re}\left(\widetilde{\mathbf{D}}_{if}^* [\widetilde{\mathbf{M}}_{if} \times \widehat{\mathbf{k}}]\right) \right), \quad (96)$$

$$e \widetilde{\mathbf{D}}_{if} \equiv \langle f | \mathbf{D}(\mathbf{k}) | i \rangle, \quad e \widetilde{\mathbf{M}}_{if} \equiv \langle f | \mathbf{M}(\mathbf{k}) | i \rangle, \quad \widehat{\mathbf{k}} \equiv \frac{\mathbf{k}}{|\mathbf{k}|}.$$

All the quantities are in the c.m. frame and averaging (summation) over initial (final) nuclear system spin states is implied. Details for calculation of the matrix elements $\langle f | \mathbf{D}(\mathbf{k}) | i \rangle$ and $\langle f | \mathbf{M}(\mathbf{k}) | i \rangle$ are given in Appendix B. When calculating the magnetic transition matrix element, only the first two terms of the $M2$ operator, Eq. (68), are retained. The last term of that formula is negligibly small for the energy range covered by the calculation while the third one is zero for the reaction considered.

To calculate the radial overlap integrals for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction, scattering- and bound-state radial wave functions have been found following the MCAS formalism described in Sec. 4. To minimize any impact of computational uncertainties in evaluation of the bound-state wave functions upon the results, we replaced these wave functions at a very long two-cluster distance ($|\mathbf{R}| > 10$ fm) by the properly renormalised Whittaker functions that correspond to experimentally known binding energies. The radial wave function for the ${}^7\text{Be}$ ground state gives rise to the rms charge radius of 2.61 fm, which is in fair agreement with the experimental value of 2.52 ± 0.3 fm [35]. The Coulomb wave functions were calculated using the CERNLIB program [36].

The main contribution to the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction amplitude at low energies is well known to result from the electric dipole transition. This is so because the quantum numbers of both the ${}^7\text{Be}$ ground state ($\frac{3}{2}^-$) and its excited ($\frac{1}{2}^-$, 429 keV) state allow the electric dipole transition to occur from the scattering s -wave. On the other hand, the operator in Eq. (47) responsible for this transition in this case is neither vanishing nor small. As a consequence, the angular distribution obtained is rather flat at very low two-cluster relative energies. Some small non-uniformity is caused by the interference between the dipole transitions from initial s and d waves. At higher energies, the $E2$ transition due to the operator given in Eq. (50) also plays a role, making the c.m. cross section asymmetric about 90° . Magnetic transitions arising from the operator in Eq. (66) are only minor contributions to the cross section as shown in Fig. 1. These observations agree with the conclusions given in Ref. [7].

The real parts of the radial overlap functions (Eq. (116) in Appendix B) for the dominant electric dipole transition for the captures to the ${}^7\text{Be}$ ground state from the relative s -state at c.m. energies of 100 keV and 1 MeV are shown in Fig. 2. For convenience, the Coulomb-phase exponential factor in Eq. (88) has been omitted. The initial- and final-state radial wave functions constructed within the MCAS scheme exhibit oscillations in the radial overlap at small (< 4 fm) distances. These wave functions and the resulting overlap function are in remarkable agreement with those given by the OCM and potential-model calculations [7]. In Fig. 2 we also compare those results with the overlaps

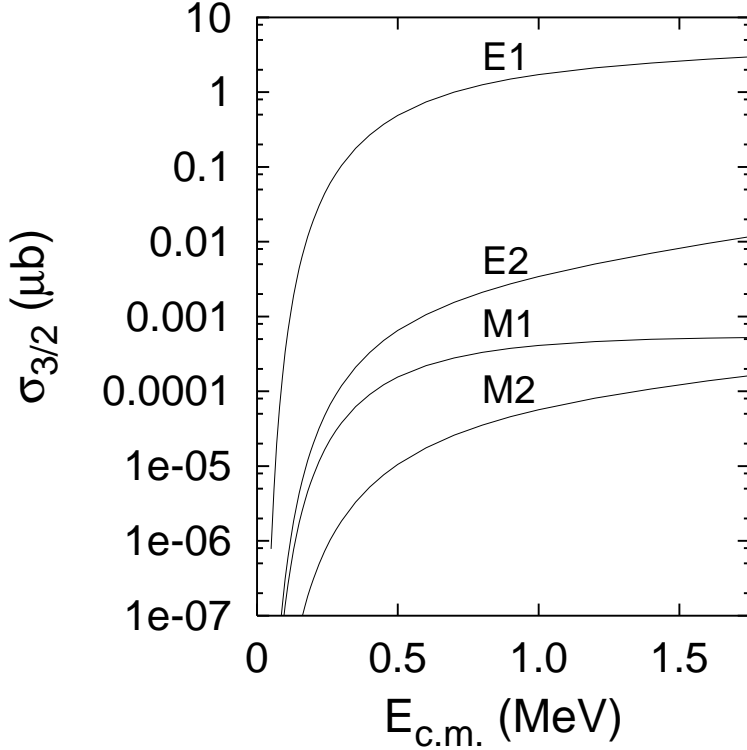


Figure 1: Multipole contributions to the total cross section for ${}^3\text{He}$ - ${}^4\text{He}$ capture leading to the ground state of ${}^7\text{Be}$.

when the initial-state radial wave function is a pure Coulomb function and the scattering wave function is that for the repulsive hard-core (RHC) model [6]. The hard sphere radius was taken as 2.8 fm. These overlap functions indicate that the photon emission occurs, on average, at very large extra-nuclear distances. However, as seen also in Fig. 2, taking into account the nuclear forces leads to a significant repulsive effect which is very important for quantitative estimates. Comparison of the curves for $E_{\text{c.m.}} = 1$ MeV and 100 keV shows the degree of spread of the EM interaction area with the decrease in the two-cluster relative energy.

In Figs. 3 and 4 respectively we show our results for the total cross section σ_{tot} and for the corresponding astrophysical S -factor which is defined as

$$S(E_{\text{c.m.}}) = E_{\text{c.m.}} \sigma_{\text{tot}}(E_{\text{c.m.}}) \exp[2\pi\eta(P)] . \quad (97)$$

Here,

$$P = \sqrt{2M_{\text{AB}}E_{\text{c.m.}}} .$$

Our calculated results reproduce the energy dependence for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ cross section and for the S -factor very well. However, the calculated cross sections are $\sim 40\%$ larger than the data. This is typical [7, 12] of both the potential and microscopic RGM cluster-model calculated results. How this overestimate depends upon the choice of the nuclear potential has been studied by Kajino [45]. He discovered that the RGM calculation with a particular nuclear force, *viz.*, the MHN interaction [46], could reproduce the absolute data for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ cross section and astrophysical S factor that were known at that time. Nevertheless, the S -factor extrapolation to the zero energy $S(0) = 0.50 \pm 0.03$ keV·barn

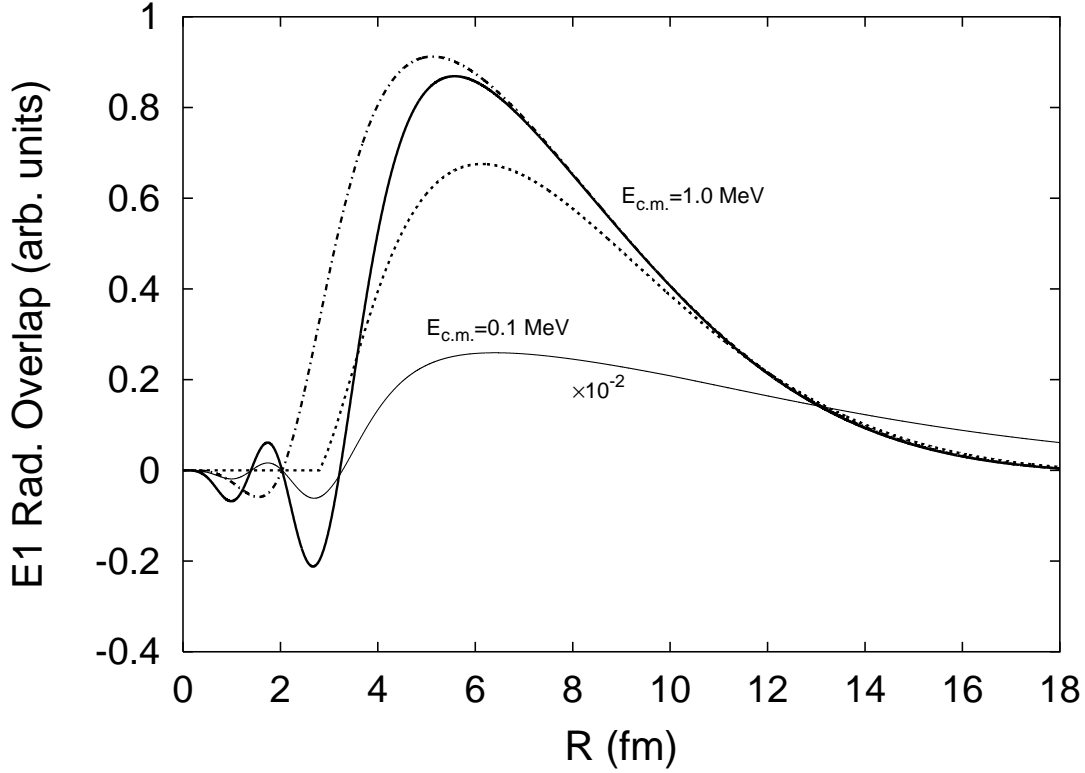


Figure 2: The real part of the radial overlap function for the electric dipole transition to the ${}^7\text{Be}$ ground state from an initial relative s -wave. The thick and thin solid curves were obtained with the scattering radial wave function defined in Eq. (88) for c.m. energies of 1 MeV and 100 keV, respectively. The dotted and dot-dashed lines are the overlaps found using the repulsive hard-core (pure Coulomb) initial-state wave functions for a c.m. energy of 1 MeV.

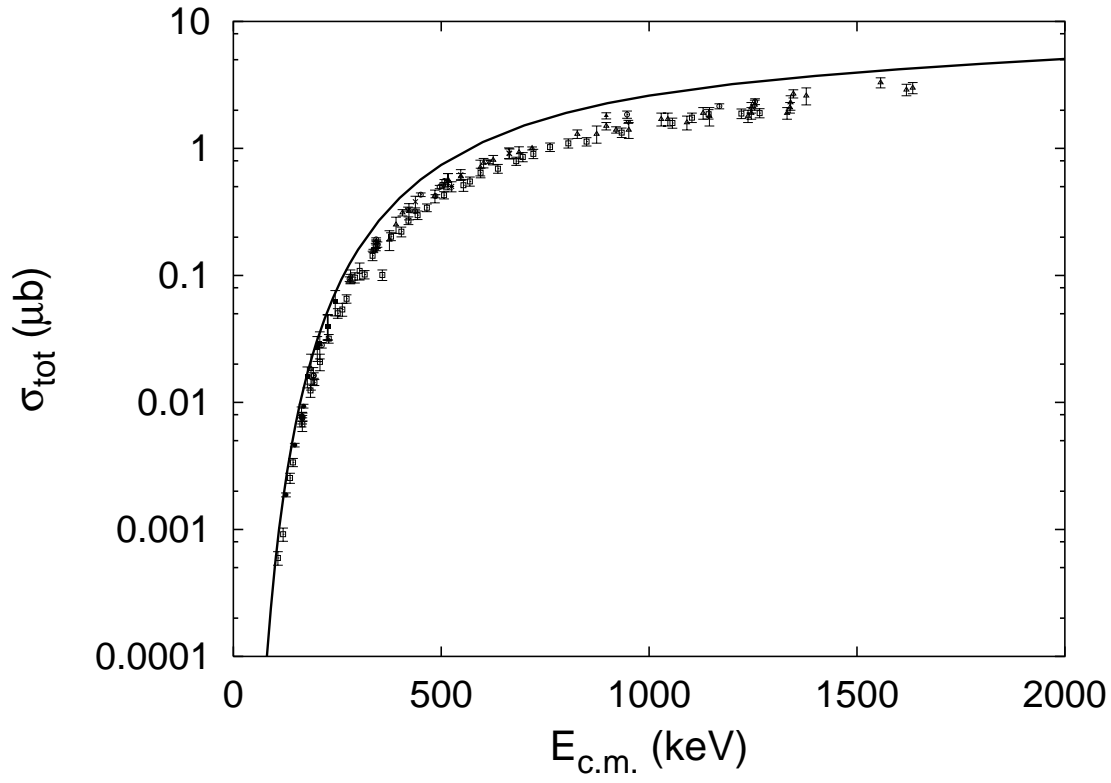


Figure 3: Total cross section of ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction. The filled circles at $E_{\text{c.m.}} = 127$, 148 and 169 keV are recent LUNA data [3]. Other experimental points were taken from Refs. [37, 38, 39, 40, 41, 42, 43, 44].

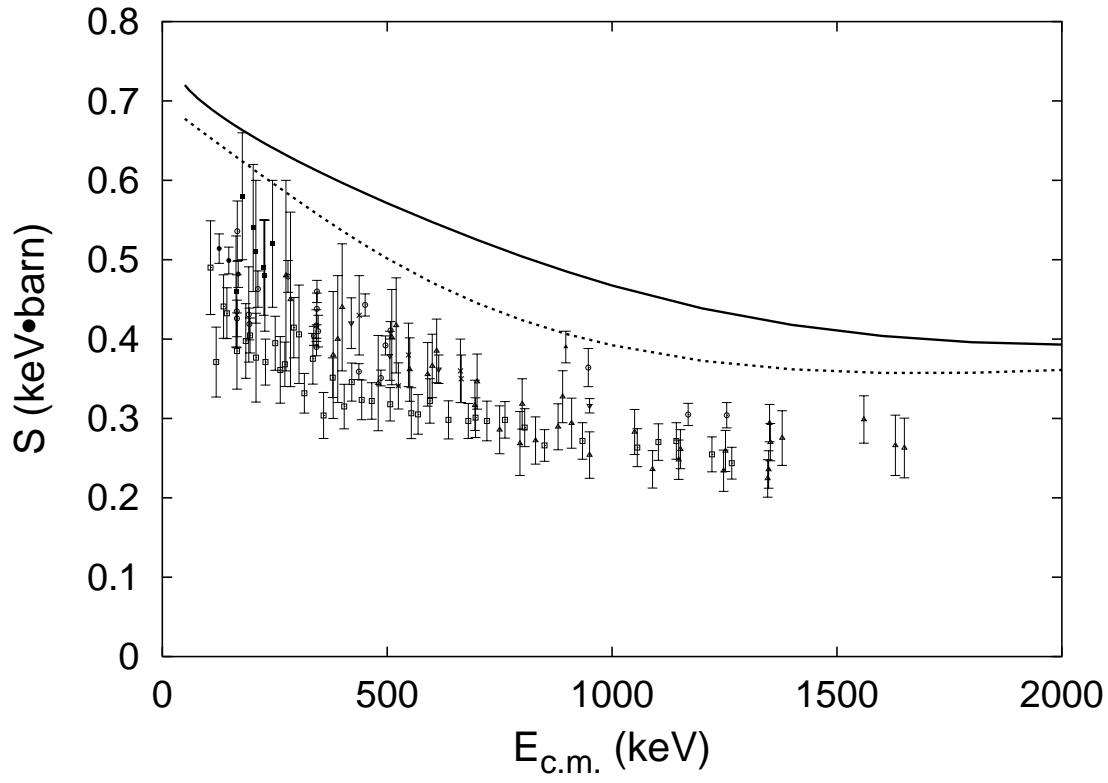


Figure 4: Astrophysical S factor for reaction ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$. The notation is as used in Fig. 3. The dotted curve displays the result obtained with the initial-state radial wave function constructed for the RHC model.

suggested in Ref. [45] is smaller than $S(0) = 0.547 \pm 0.017$ keV·barn given by the latest R -matrix fit to all current data, including that from the LUNA experiment [3]. Evidently, the two-cluster $\alpha + {}^3\text{He}$ configuration does not exhaust all the possibilities for the ${}^7\text{Be}$ bound state [19]. In principle, that could explain why the magnitude of the calculated cross section is larger than experiment. However, it was indicated [16] that extension of the two-cluster model space for ${}^7\text{Be}$ by inclusion of the $p + {}^6\text{Li}$ channel might even make the agreement between the theory and experiment worse.

In contrast, the variational Monte Carlo calculation [15] made with the two-body wave functions constructed from nucleon-nucleon interactions found an S factor that lay significantly below the experimental values for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction. In particular, the value of $S(0) = 0.40$ keV·barn has been extrapolated. But this approach, without the inclusion of multi-nucleon forces, was not able to reproduce satisfactorily the ${}^7\text{Be}$ bound-state levels and the binding energies for the separated ${}^3\text{He}$ and ${}^4\text{He}$ fragments. In addition, phenomenological procedures had to be used to construct the two-cluster (7-body) scattering wave functions. With respect to this last point, note that recent few-body techniques have been developed to deal with this difficult problem [47].

Though our treatment is not of the “ab initio” type, both the initial- and final-state radial wave functions have been constructed using the same two-body potential [19]; that potential with which the ${}^7\text{Li}$ two-cluster binding energy and a number of resonance positions and widths were determined. This potential also gave the $\alpha + {}^3\text{He}$ system resonance parameters listed in Table 1 and they, too, are in good agreement with experiment. In this table, the state positions are given relative to $\alpha - {}^3\text{He}$ breakup threshold and all units are MeV. Nevertheless, the ${}^7\text{Be } \frac{1}{2}^-$ excited state appears to be too weakly bound

Table 1: Spectrum of states for ${}^7\text{Be}$ as $\alpha + {}^3\text{He}$ system.

J^π	Level position (width)	Exp. level position (width)
$\frac{3}{2}^-$	-1.53	-1.59
$\frac{1}{2}^-$	-0.84	-1.16
$\frac{7}{2}^-$	3.07 (0.180)	2.98 (0.175)
$\frac{5}{2}^-$	5.09 (1.2)	5.14 (1.2)

in our calculations. This affects the corresponding bound-state wave function behaviour in the EM interaction region. As a result, we obtained the branching ratio (ratio of cross section for capture into the $\frac{1}{2}^-$ state to that for transition into the ground state) equal to 0.5. That is slightly above the average experimental value of ~ 0.45 . Our calculated ratio essentially is independent of the energy in complete agreement with both experimental data and other calculations.

Using the MCAS two-cluster potential [19] the existing data on ${}^3\text{H}(\alpha, \alpha){}^3\text{H}$ and ${}^4\text{He}({}^3\text{He}, {}^3\text{He}){}^4\text{He}$ scattering reactions for the c.m. energy range between 2 and 7 MeV is well reproduced. It also gives good representation of the low lying states in the mass-7 compound nuclei that can be formed. When choosing the “optimal” potential parameters in that treatment, there were no evaluation of the nuclear scattering phase shifts at lower energies. Comparison of our result for the astrophysical S factor with that obtained in case of replacement of the MCAS initial-state radial wave function by the RHC scattering wave function (see Fig. 4) suggests that the agreement of our calculations with experimen-

tal data could be somewhat improved by correction of the MCAS potential parameters for better description of nuclear scattering at very low (below the lowest resonance) energies. Though, one should keep in mind, when performing this comparison, that the RHC initial-state radial wave function is not properly normalized.

Adjustment of the potential given in Ref. [19] to find a better fit to the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ radiative capture data has not been attempted in this study as our primary goal has been to present a comprehensive formalism for the treatment of radiative-capture reactions within the MCAS two-cluster picture. We have made a first application of such an approach, but only in a simplified test case. The general approach has been developed, however, with a view of application to radiative-capture processes involving more complex nuclei. For this reason we paid attention to particular aspects of the EM transition such as the role of finite fragment sizes and collective (rotation and vibration) excitations. Nevertheless, with the MCAS scheme and without any extra adjustment of the given potential model we could reproduce the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ cross section as well as other theoretical models, be they microscopic or based on a potential specifically constructed to describe the capture cross section.

6 Conclusions and outlook

A formalism has been presented to describe direct radiative-capture reactions at low energies within an extended two-cluster potential model. Unlike customary potential cluster models in which the interacting nuclear fragments are usually treated as point-like objects, the EM current operator has been used so that finite sizes of clusters and their intrinsic excitations are taken into account. Inherent in the approach is the explicit gauge independence and reproduction of the low-energy behaviour of the EM transition amplitude, which follows from the current conservation *viz.*, fulfilment of the Siegert theorem [22] for electric transitions in the long-wavelength limit. These properties are ensured by extension [21, 25] of the Siegert theorem which served as a starting point to build the photon-emission operator. The formalism was used to construct the electric and magnetic operators responsible for low-energy EM transitions in which the intrinsic parities of clusters remain unchanged. Starting with a microscopic single-nucleon current model, construction of these operators lead to expressions in terms of macroscopic quantities which refer to fragment EM static properties or intrinsic transitions such as the cluster mean-square radius and magnetic dipole and electric quadrupole moments. Such a feature allows one to perform calculations with wave functions of a nuclear model including collective-type coupled-channel dynamics.

Indeed, it has been shown how the photon-emission operator can be utilised to calculate the cross section of the low-energy radiative-capture process (72) using multi-channel radial wave functions for the initial (scattering) and final (bound) states. One of the goals of this study was to match the constructed EM current with the MCAS coupled-channel approach [17, 19] developed for description of nuclear scattering. We have presented the explicit construction for the scattering radial wave function based on the MCAS formalism. The corresponding expression is given for a separable two-body potential of an arbitrary finite rank in terms of the potential form factors, the MCAS T matrix and the regular and irregular Coulomb functions. This wave function construction is general in the sense that it is given for a two-cluster nuclear potential with non-central forces. Possible

inelastic channels are taken into account as is Coulomb distortion. Detailed formulae for the matrix elements of the EM operator between the constructed nuclear states have also been given. Thus, applicability of the MCAS theory has been extended to cover nuclear radiative-capture reactions including ones of astrophysical relevance.

Finally, we have applied the formalism to calculate the total cross section and astrophysical S factor for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ radiative capture at low energies. This reaction is strongly dominated by the electric dipole transition, with the effects due to cluster sizes and intrinsic structure being negligible. In addition, there are no inelastic channels in the $\alpha + {}^3\text{He}$ system, while the elastic channels are uncoupled. Thus many features of the developed formalism are not required in analysing this reaction. However, not only does this application serve our purpose, the reaction itself is important from the astrophysical point of view. For the latter reason it has been extensively studied both theoretically (in different models) and experimentally. Our calculation aimed at establishing the level at which the MCAS two-body potential scheme is able to reproduce the observed cross-section (and S -factor) energy dependence at low energy without adjustment of potential parameters. To obtain the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ cross section, we used the MCAS potential [19] with parameters providing the best fit to state positions and resonance widths for the mirror two-fragment system $\alpha + {}^3\text{H}$. The energy dependence of our calculated cross section is in a fair agreement with experiment and with that found using other theoretical models. The calculated S -factor is about 40 % above the experimental data, a result which is also typical of calculations performed within other models, in particular those based upon the RGM.

As shown in Ref. [45], better agreement with experiment can be achieved if one employs a two-body potential with parameters tuned to describe fairly well not only the two-fragment bound-state levels and resonance parameters, but also the static EM properties of the final nucleus. However, the corresponding potential analysis should also include the correction to the nucleus static magnetic moment due to the interaction current contribution. That comes from nonlocalities (dependence on the velocity) of the two-cluster potential. Besides the “one-body” EM operator discussed in the present paper, a procedure for constructing the interaction current for the nuclear two-fragment system is needed. That is work in progress.

The formalism presented herein, based on the construction of the EM operator and the wave functions built using the MCAS approach [17, 19], establishes a single potential framework for a unified description of low-energy nuclear scattering, radiative-capture reactions, and of static EM properties of nuclei, in the case that they can be treated as two-cluster systems. It is our intention to consider scattering and capture reactions involving more complex nuclei than the relatively simple ${}^{3,4}\text{He}$. In those future studies, we expect that the role of cluster sizes and intrinsic excitations in forming the energy dependence for the corresponding astrophysical S factors will be of great importance.

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A Appendix: Details of the magnetic operator

Assume that nuclear system dynamics is described by a Hamiltonian

$$H = H_A + H_B + H_{AB} , \quad (98)$$

with H_A (H_B) being the subsystem A (B) intrinsic Hamiltonian, and H_{AB} describing the two-cluster relative motion. Let us obtain the k -order correction to the leading-order contribution (62) in the part of the magnetic operator (34), which gives rise to transitions without changing the parity of the cluster intrinsic states.

First, consider the joint contribution of order $\mu_N k$ given by the first and sixth terms of Eq. (54), for which we have

$$\mathbf{X}_{16}(\lambda \mathbf{k}) = -i\lambda \frac{M_B}{M} \left((\mathbf{k} \mathbf{R}) \mathbf{m}_A^{\text{orb}}(0) + \mathbf{R} \times \frac{1}{2} \sum_{\alpha=1}^A \hat{e}_\alpha \{ \mathbf{v}_\alpha^A, \mathbf{k} \mathbf{r}_\alpha^A \} \right) . \quad (99)$$

If the intrinsic potential for the cluster A is local, then the relationship

$$\mathbf{v}_\alpha^A = i \left[H_A, \mathbf{r}_\alpha^A \right] \quad (100)$$

is equivalent to the one given for the quantities \mathbf{v}_α^A and \mathbf{p}_α^A by Eq. (58). However, Eq. (100) can be considered as a definition of operator \mathbf{v}_α^A which enters Eq. (54) in a more general case (see Ref. [23], p. 393).

Using Eq. (100), we get the relationship

$$\begin{aligned} i [H_A, \mathbf{d}_A(\lambda \mathbf{k})] &= i \sum_{\alpha=1}^A \hat{e}_\alpha \left[H_A, \mathbf{r}_\alpha^A e^{-i\lambda \mathbf{k} \mathbf{r}_\alpha^A} \right] \\ &= \sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{v}_\alpha^A e^{-i\lambda \mathbf{k} \mathbf{r}_\alpha^A} + i \sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{r}_\alpha^A \left[H_A, e^{-i\lambda \mathbf{k} \mathbf{r}_\alpha^A} \right] \\ &= \mathbf{v}_A(0) - i\lambda \sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{v}_\alpha^A (\mathbf{k} \mathbf{r}_\alpha^A) - i\lambda \sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{r}_\alpha^A (\mathbf{k} \mathbf{v}_\alpha^A) + O(\mu_N k^2) . \end{aligned} \quad (101)$$

On the other hand, we can write for the quantities $\mathbf{d}_{A,B}$

$$\mathbf{d}_{A,B}(\lambda \mathbf{k}) = \mathbf{d}_{A,B}(0) - i\lambda \mathbf{d}_{A,B}^{(1)}(\mathbf{k}) + O(k^2) , \quad (102)$$

$$\begin{aligned} d_{A,B}^{(1)}(\mathbf{k}) &= \frac{1}{3} \sum_l \left(k_l t_{jl}^{A,B} + e k_j Z_{A,B} r_{A,B}^2 \right) , \\ (j, l &= 1, 2, 3) \end{aligned} \quad (103)$$

where $t_{jl}^{A,B}$ is the cluster A, B electric quadrupole operator defined by Eq. (49), and

$$r_{A,B}^2 = \frac{1}{Z_{A,B}} \sum_{\xi=1}^{A,B} \hat{e}_\xi \left[\mathbf{r}_\xi^{A,B} \right]^2 \quad (104)$$

is the square charge radius operator for the corresponding subsystem. Due to the parity selection, the contributions from $\mathbf{v}_A(0)$ and $\mathbf{d}_A(0)$ vanish for transitions between same-parity H_A eigenstates. Hence, owing to Eqs. (101) and (102), we can write

$$\sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{v}_\alpha^A (\mathbf{k} \mathbf{r}_\alpha^A) = - \sum_{\alpha=1}^A \hat{e}_\alpha \mathbf{r}_\alpha^A (\mathbf{k} \mathbf{v}_\alpha^A) + i \left[H_A, \mathbf{d}_A^{(1)}(\mathbf{k}) \right] . \quad (105)$$

omitting higher-order contributions. The latter relationship allows us to write

$$\begin{aligned} \frac{1}{2} \sum_{\alpha=1}^A \hat{e}_{\alpha} \{ \mathbf{v}_{\alpha}^A, \mathbf{k} \mathbf{r}_{\alpha}^A \} &= \frac{1}{2} \sum_{\alpha=1}^A \hat{e}_{\alpha} (\mathbf{k} \mathbf{r}_{\alpha}^A) \mathbf{v}_{\alpha}^A - \frac{1}{2} \sum_{\alpha=1}^A \hat{e}_{\alpha} \mathbf{r}_{\alpha}^A (\mathbf{k} \mathbf{v}_{\alpha}^A) + \frac{i}{2} [H_A, \mathbf{d}_A^{(1)}(\mathbf{k})] \\ &= -\frac{1}{2} \sum_{\alpha=1}^A \hat{e}_{\alpha} \mathbf{k} \times [\mathbf{r}_{\alpha}^A \times \mathbf{v}_{\alpha}^A] + \frac{i}{2} [H_A, \mathbf{d}_A^{(1)}(\mathbf{k})] . \end{aligned} \quad (106)$$

Substituting Eq. (106) into Eq. (99) and omitting longitudinal terms i.e. those proportional to vector \mathbf{k} , we get

$$\mathbf{X}_{16}(\lambda \mathbf{k}) = -i \frac{3}{2} \lambda \frac{M_B}{M} (\mathbf{k} \mathbf{R}) \mathbf{m}_A^{\text{orb}}(0) + \frac{1}{2} \lambda \frac{M_B}{M} \mathbf{R} \times [H_A, \mathbf{d}_A^{(1)}(\mathbf{k})] . \quad (107)$$

Analogously, the joint contribution from the second and seventh terms of Eq. (54) is given by

$$\mathbf{X}_{27}(\lambda \mathbf{k}) = i \frac{3}{2} \lambda \frac{M_A}{M} (\mathbf{k} \mathbf{R}) \mathbf{m}_B^{\text{orb}}(0) - \frac{1}{2} \lambda \frac{M_A}{M} \mathbf{R} \times [H_B, \mathbf{d}_B^{(1)}(\mathbf{k})] . \quad (108)$$

Since transitions between states of the same parity cannot be made with the operators $\mathbf{d}_{A,B}(0)$, the 4th and 5th terms in Eq. (54) of the lowest (first) order in q , provide the contribution

$$\mathbf{X}_{45}(\lambda \mathbf{k}) = i \lambda \frac{M_B \mathbf{d}_A^{(1)}(\mathbf{k}) - M_A \mathbf{d}_B^{(1)}(\mathbf{k})}{M} \times \mathbf{V} . \quad (109)$$

Substitution of Eqs. (107) – (109) and the same-order contribution given by Eqs. (56) into Eq. (34) and taking into account the spin contribution coming from Eq. (60) gives

$$\begin{aligned} -i \mathbf{M}^{M2}(\mathbf{k}) &= -i (\mathbf{k} \mathbf{R}) \left(\frac{M_B}{M} \boldsymbol{\mu}^A - \frac{M_A}{M} \boldsymbol{\mu}^B \right) \\ &+ \frac{1}{6} \mathbf{R} \times \left(\frac{M_B}{M} [H_A, \mathbf{d}_A^{(1)}(\mathbf{k})] - \frac{M_A}{M} [H_B, \mathbf{d}_B^{(1)}(\mathbf{k})] \right) \\ &- \frac{i}{3} \frac{M_B \mathbf{d}_A^{(1)}(\mathbf{k}) - M_A \mathbf{d}_B^{(1)}(\mathbf{k})}{M} \times \mathbf{V} - i \frac{e}{2m_N} C_{M2} \{ \mathbf{L}, (\mathbf{k} \mathbf{R}) \} , \end{aligned} \quad (110)$$

$$C_{M2} = \frac{1}{3A_{\text{tot}} M} \left(\frac{M_B^2}{M_A} Z_A - \frac{M_A^2}{M_B} Z_B \right) . \quad (111)$$

B Appendix: EM transition matrix elements

In this Appendix, we present the detailed formulae for the transition matrix elements of the EM operator constructed in Sec. 3 between the initial (scattering) and final (bound) states described in Sec. 4 for the radiative capture process, Eq. (72). The notation is as used in those sections.

Spherical components of the matrix elements of the operators given in Eqs. (46) and (66) between the states specified in Eqs. (73) and (74) can be written as

$$\begin{aligned} \langle f | D_{\xi}(\mathbf{k}) | i \rangle &= \frac{E_i^{\text{int}} - E_f^{\text{int}}}{E_i - E_f} \left\{ e C_{E1} [R_{\xi}]_{if} - \frac{i}{6} (-1)^{\kappa} k_{-\kappa} \left([t_{\xi\kappa}^A]_{if} + [t_{\xi\kappa}^B]_{if} \right. \right. \\ &\quad \left. \left. + e C_{E2} [T_{\xi\kappa}^{\text{AB}}]_{if} \right) \right\} , \end{aligned} \quad (112)$$

$$\begin{aligned}
\langle f | M_\xi(\mathbf{k}) | i \rangle &= [\mu_\xi^A]_{if} + [\mu_\xi^B]_{if} + \mu_N C_{M1} [L_\xi]_{if} \\
&- i(-1)^\kappa k_{-\kappa} \left(\frac{M_B}{M} [\mu_\xi^A R_\kappa]_{if} - \frac{M_A}{M} [\mu_\xi^B R_\kappa]_{if} + \mu_N C_{M2} [\{L_\xi, R_\kappa\}]_{if} \right), \quad (113)
\end{aligned}$$

$$(\xi, \kappa = -1, 0, 1)$$

where, for simplicity, we ignore the last two terms of expression (68), which either do not contribute at all, or are negligibly small for the energy range covered by our calculation of this amplitude presented in Sec. 5. Then, the matrix elements

$$[\mathcal{O}]_{if} \equiv \langle \Phi_{J_C}^C J_C m_C | \mathcal{O} | \Psi_{\mathbf{P}}^{(+)}; \phi_{J_A}^A J_A m_A; \phi_{J_B}^B J_B m_B \rangle, \quad (114)$$

which enter Eqs. (112) and (113), can be written as

$$[\mathcal{O}]_{if} = \frac{\omega_L}{\sqrt{4\pi}} \sum_{a, a', a'', J} C(L 0 J_A m_A J_A''' m_A J_B m_B J, m_A + m_B) \hat{J} \hat{L} [\mathcal{O}]_{a' a'' a}^{J J_C m_C}, \quad (115)$$

where

$$\hat{j} \equiv \sqrt{2j+1}$$

and

$$\omega_L = \begin{cases} 1 & \text{if A is projectile, and B is target} \\ (-1)^L & \text{otherwise.} \end{cases}$$

Here, the angular-momentum quantization axis is taken to be in the direction of the momentum \mathbf{P} . Denoting the radial overlap integral as

$$I_{a' a'' a}^{J_C J(\nu)} \equiv \int \Phi_{J_C a'}^C(R) \Psi_{a'' a}^{J(+)}(R) R^{2+\nu} dR, \quad (116)$$

for the angular-momentum coupling scheme defined by Eq. (75), we have

$$\begin{aligned}
[R_\xi]_{a' a'' a}^{J J_C m_C} &= \delta_{J'_A J''_A} \delta_{J'_B J''_B} (-1)^{J+J'+J''+L'+J'_A+J'_B} \hat{J}' \hat{J}'' \hat{L}'' \langle L'' 0 1 0 | L' 0 \rangle \\
&\times \langle J, m_A + m_B, 1, \xi | J_C m_C \rangle \left\{ \begin{matrix} L'' & J'_A & J'' \\ J' & 1 & L' \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & 1 & J' \end{matrix} \right\} I_{a' a'' a}^{J_C J(1)}, \quad (117)
\end{aligned}$$

$$\begin{aligned}
[t_{\xi\kappa}^A]_{a' a'' a}^{J J_C m_C} &= \sqrt{\frac{3}{2}} \delta_{L' L''} \delta_{J'_B J''_B} (-1)^{J-L'+2J'_A+J''_A+J'_B} \hat{J}' \hat{J}'' \langle 1\xi 1\kappa | 2, \xi + \kappa \rangle \\
&\times \langle J, m_A + m_B, 2, \xi + \kappa | J_C m_C \rangle \left\{ \begin{matrix} J''_A & L' & J'' \\ J' & 2 & J'_A \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & 2 & J' \end{matrix} \right\} \\
&\times \langle \phi_{J'_A}^A || \mathcal{Q}_A || \phi_{J''_A}^A \rangle I_{a' a'' a}^{J_C J(0)}, \quad (118)
\end{aligned}$$

$$\begin{aligned}
[t_{\xi\kappa}^B]_{a' a'' a}^{J J_C m_C} &= \sqrt{\frac{3}{2}} \delta_{J' J''} \delta_{L' L''} \delta_{J'_A J''_A} (-1)^{2J'_B+J''_B-J'-J_C} \langle 1\xi 1\kappa | 2, \xi + \kappa \rangle \\
&\times \langle J, m_A + m_B, 2, \xi + \kappa | J_C m_C \rangle \left\{ \begin{matrix} J''_B & J' & J \\ J_C & 2 & J'_B \end{matrix} \right\} \langle \phi_{J'_B}^B || \mathcal{Q}_B || \phi_{J''_B}^B \rangle I_{a' a'' a}^{J_C J(0)}, \quad (119)
\end{aligned}$$

$$\begin{aligned}
[T_{\xi\kappa}^{\text{AB}}]_{a'a''a}^{JJ_C m_C} &= \sqrt{6} \delta_{J'_A J''_A} \delta_{J'_B J''_B} (-1)^{J+J'+J''+L'+J'_A+J'_B} \widehat{J'} \widehat{J''} \widehat{L''} \langle 1\xi 1\kappa \mid 2, \xi + \kappa \rangle \\
&\times \langle J, m_A + m_B, 2, \xi + \kappa \mid J_C m_C \rangle \left\{ \begin{matrix} L'' & J'_A & J'' \\ J' & 2 & L' \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & 2 & J' \end{matrix} \right\} \\
&\times \langle L'' 0 2 0 \mid L' 0 \rangle I_{a'a''a}^{J_C J^{(2)}}, \tag{120}
\end{aligned}$$

$$\begin{aligned}
[\mu_{\xi}^{\text{A}}]_{a'a''a}^{JJ_C m_C} &= \delta_{L' L''} \delta_{J'_B J''_B} (-1)^{J-L'+2J'_A+J''_A+J'_B} \widehat{J'} \widehat{J''} \langle J, m_A + m_B, 1, \xi \mid J_C m_C \rangle \\
&\times \left\{ \begin{matrix} J''_A & L' & J'' \\ J' & 1 & J'_A \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & 1 & J' \end{matrix} \right\} \langle \phi_{J'_A}^{\text{A}} \parallel \mathcal{M}_A \parallel \phi_{J''_A}^{\text{A}} \rangle I_{a'a''a}^{J_C J^{(0)}}, \tag{121}
\end{aligned}$$

$$\begin{aligned}
[\mu_{\xi}^{\text{B}}]_{a'a''a}^{JJ_C m_C} &= \delta_{J' J''} \delta_{L' L''} \delta_{J'_A J''_A} (-1)^{2J'_B+J''_B-J'-J_C-1} \langle J, m_A + m_B, 1, \xi \mid J_C m_C \rangle \\
&\times \left\{ \begin{matrix} J''_B & J' & J \\ J_C & 1 & J'_B \end{matrix} \right\} \langle \phi_{J'_B}^{\text{B}} \parallel \mathcal{M}_B \parallel \phi_{J''_B}^{\text{B}} \rangle I_{a'a''a}^{J_C J^{(0)}}, \tag{122}
\end{aligned}$$

$$\begin{aligned}
[L_{\xi}]_{a'a''a}^{JJ_C m_C} &= \delta_{J'_A J''_A} \delta_{J'_B J''_B} \delta_{L' L''} (-1)^{J+J'+J''+L'+J'_A+J'_B} \widehat{J'} \widehat{J''} \widehat{L'} \sqrt{L'(L'+1)} \\
&\times \langle J, m_A + m_B, 1, \xi \mid J_C m_C \rangle \left\{ \begin{matrix} L'' & J'_A & J'' \\ J' & 1 & L' \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & 1 & J' \end{matrix} \right\} I_{a'a''a}^{J_C J^{(0)}}, \tag{123}
\end{aligned}$$

$$\begin{aligned}
[\mu_{\xi}^{\text{A}} R_{\kappa}]_{a'a''a}^{JJ_C m_C} &= \delta_{J'_B J''_B} (-1)^{J+J'+J'_B} \widehat{J'} \widehat{J''} \widehat{L''} \langle L'' 0 1 0 \mid L' 0 \rangle \\
&\times \langle \phi_{J'_A}^{\text{A}} \parallel \mathcal{M}_A \parallel \phi_{J''_A}^{\text{A}} \rangle I_{a'a''a}^{J_C J^{(1)}} \sum_g \widehat{g} \langle 1\xi 1\kappa \mid g, \xi + \kappa \rangle \\
&\times \langle J, m_A + m_B, g, \xi + \kappa \mid J_C m_C \rangle \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & g & J' \end{matrix} \right\} \left\{ \begin{matrix} 1 & 1 & g \\ L' & J'_A & J' \\ L'' & J''_A & J'' \end{matrix} \right\}, \tag{124}
\end{aligned}$$

$$\begin{aligned}
[\mu_{\xi}^{\text{B}} R_{\kappa}]_{a'a''a}^{JJ_C m_C} &= \delta_{J'_A J''_A} (-1)^{J''+L'+J'_A} \widehat{J'} \widehat{J''} \widehat{L''} \langle L'' 0 1 0 \mid L' 0 \rangle \\
&\times \langle \phi_{J'_B}^{\text{B}} \parallel \mathcal{M}_B \parallel \phi_{J''_B}^{\text{B}} \rangle \left\{ \begin{matrix} L'' & J'_A & J'' \\ J' & 1 & L' \end{matrix} \right\} I_{a'a''a}^{J_C J^{(1)}} \\
&\times \sum_g (-1)^g \widehat{g} \langle 1\xi 1\kappa \mid g, \xi + \kappa \rangle \langle J, m_A + m_B, g, \xi + \kappa \mid J_C m_C \rangle \left\{ \begin{matrix} 1 & 1 & g \\ J' & J'_B & J_C \\ J'' & J''_B & J \end{matrix} \right\} \tag{125}
\end{aligned}$$

$$\begin{aligned}
[\{L_{\xi}, R_{\kappa}\}]_{a'a''a}^{JJ_C m_C} &= \delta_{J'_A J''_A} \delta_{J'_B J''_B} (-1)^{J+J'+J''+L''+J'_A+J'_B} \widehat{J'} \widehat{J''} \widehat{L''} \langle L'' 0 1 0 \mid L' 0 \rangle I_{a'a''a}^{J_C J^{(1)}} \\
&\times \sum_g \widehat{g} \langle 1\xi 1\kappa \mid g, \xi + \kappa \rangle \langle J, m_A + m_B, g, \xi + \kappa \mid J_C m_C \rangle \left\{ \begin{matrix} L'' & J'_A & J'' \\ J' & g & L' \end{matrix} \right\} \left\{ \begin{matrix} J'' & J'_B & J \\ J_C & g & J' \end{matrix} \right\} \\
&\times \left((-1)^g \widehat{L'} \sqrt{L'(L'+1)} \left\{ \begin{matrix} 1 & 1 & g \\ L' & L'' & L' \end{matrix} \right\} + \widehat{L''} \sqrt{L''(L''+1)} \left\{ \begin{matrix} 1 & 1 & g \\ L' & L'' & L'' \end{matrix} \right\} \right), \tag{126}
\end{aligned}$$

where quantities $\langle \phi_{J'_{\text{A,B}}}^{\text{A,B}} \parallel \mathcal{M}_{\text{A,B}} \parallel \phi_{J''_{\text{A,B}}}^{\text{A,B}} \rangle$ and $\langle \phi_{J'_{\text{A,B}}}^{\text{A,B}} \parallel \mathcal{Q}_{\text{A,B}} \parallel \phi_{J''_{\text{A,B}}}^{\text{A,B}} \rangle$ are the reduced matrix elements of the magnetic dipole and electric quadrupole operators for cluster A, B. In a

particular case of transitions that do not change the fragment intrinsic states ($\phi_{J'_{A,B}}^{A,B} = \phi_{J''_{A,B}}^{A,B}$), they are related to the static magnetic dipole ($\mu_{A,B}$) and electric quadrupole ($Q_{A,B}$) moments of the nucleus A, B through the corresponding definitions [28]

$$\begin{aligned} \frac{\langle J'_{A,B} J'_{A,B} \ 1 \ 0 \mid J'_{A,B} J'_{A,B} \rangle}{\widehat{J'_{A,B}}} \langle \phi_{J'_{A,B}}^{A,B} \parallel \mathcal{M}_{A,B} \parallel \phi_{J'_{A,B}}^{A,B} \rangle &= \mu_N \mu_{A,B} \\ \frac{\langle J'_{A,B} J'_{A,B} \ 2 \ 0 \mid J'_{A,B} J'_{A,B} \rangle}{\widehat{J'_{A,B}}} \langle \phi_{J'_{A,B}}^{A,B} \parallel \mathcal{Q}_{A,B} \parallel \phi_{J'_{A,B}}^{A,B} \rangle &= e Q_{A,B} . \end{aligned} \quad (127)$$

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